Journal of Information Processing Vol.24 No.4 660–668 (July 2016)

[DOI: 10.2197/ipsjjip.24.660]

Regular Paper

Fluid-Based Analysis for Understanding TCP Performance on Scale-Free Structure

Yusuke Sakumoto[1,a)] Hiroyuki Ohsaki[2,b)]

Received: November 16, 2015, Accepted: April 5, 2016

Abstract: Scale-free structure is one of the most notable properties of the Internet as a complex network. Many researchers have investigated the end-to-end performance (e.g., throughput, packet loss probability, and round-trip time between source/destination nodes) of TCP congestion control mechanisms, but the impact of the scale-free structure on the TCP performance has not been fully understood. In this paper, we analyze the TCP performance on a scale-free tree whose strength of the scale-free property can be adjusted by a parameter. A scale-free tree represents the communication kernel for investigating a scale-free network since TCP mainly transmits packets on a shortest path between TCP source/destination nodes, and most shortest paths are included in the scale-free tree. Our numerical results show that the scale-free structure of a network improves the TCP performance, and that such performance improvement is caused by a reduction in the average path length and also a reduction of the traffic intensity at the bottleneck link. Furthermore, we confirm the validity of our analysis through a comparison with an optimization-based analysis.

Keywords: large-scale network, scale-free structure, end-to-end performance, TCP, fluid-based analysis

1. Introduction

The Internet is extremely large in size, and has the common properties (e.g., scale-free property, small-world property) of complex networks [1]. In order to realize an effective traffic delivery on the Internet, we should understand how the properties of complex networks affects performance of the traffic delivery.

TCP is transmitting most of traffic on the Internet, and the end-to-end performance of TCP congestion control mechanisms has been studied in a large number of papers (see, e.g., Refs. [2], [3], [4], [5], [6]). Typical examples of the end-to-end performance are the throughput, the packet loss probability, and the round-trip time, shown in **Fig. 1**. Hereinafter, we collectively call them *the TCP performance*. Most of the previous studies ignore the properties of complex networks in evaluation, and the impact of the complex network properties on the TCP performance has not been figured out sufficiently.

Scale-free structure is the most notable characteristic of the Internet as a complex network. If a network has the scale-free structure, the degree distribution of nodes in the network follows the power law. Power law degree distributions generate a few hub nodes, which have an extremely large degree. In such a network, many of the shortest paths among nodes are presumably included in links connected to a hub node, so a serious traffic concentration would occur in the vicinity of the hub nodes. Hence, the scale-free structure should act in the direction of degrading throughput and increasing packet loss probability between source/destination nodes. In addition, it is known that the scale-free structure makes the average of shortest path lengths among nodes smaller [7]. Hence, the scale-free structure should act in the direction of decreasing the average of round-trip times between source/destination nodes. According to the well-known TCP analysis [8], a smaller round-trip time conduces a better TCP throughput. However, to the best of my knowledge, no one has sufficiently clarified how the above acting of the scale-free structure affects the TCP performance.

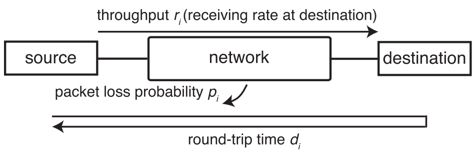

Fig. 1 TCP performance.

In this paper, we investigate the impact of the scale-free structure on the TCP performance using fluid-based analysis. In this investigation, we use a scale-free tree as a network topology. A scale-free tree represents the communication kernel for investigating a scale-free network since TCP mainly transmits packets on a shortest path between TCP source/destination nodes, and most of shortest paths are included in the scale-free tree [9]. We generate scale-free trees with the method [10], which can adjust the strength of the scale-free property by a parameter. In order to clarify statistical characteristic of the TCP performance in the scale-free structure, we focus on TCP flows passing through a bottleneck link in a scale-free tree, and derive the probability distributions of the TCP performance using the analytic method [11] based on fluid-flow approximation of TCP congestion control. Through numerical examples of our analysis, we clarify the im-

1 Graduate School of System Design, Tokyo Metropolitan University, Hino, Tokyo 191–0065, Japan
2 Graduate School of Science and Technology, Kwansei Gakuin University, Sanda, Hyogo 669–1337, Japan
a) sakumoto@tmu.ac.jp
b) ohsaki@kwansei.ac.jp

pact of the scale-free structure on the TCP performance. Furthermore, we confirm the validity of our analysis through a comparison with an optimization-based analysis [12].

The main contribution of this paper is divided into two parts. In the first part, we derive the probability distributions of the TCP performance on scale-free trees. Fekete et al. has conducted a pioneering work for the analysis of the TCP performance on scale-free trees. However, they did not consider the heterogeneity of TCP flows in term of shortest path length, and derived the mean of the throughput with the same shortest path lengths. If shortest path lengths of TCP flows are different, the performance of TCP flows vary with its shortest path length. Hence, to discuss the mean of the TCP performance with considering the heterogeneity, we should derive firstly the probability distributions. Since it is difficult to derive the probability distributions of the TCP performance on large-scale trees, we focus on TCP flows passing through a bottleneck link in a scale-free tree, and succeed in the derivation of the probability distributions that can explain the qualitative property of the TCP performance. In the second part, we clarify the impact of the scale-free structure on the TCP performance. Ohsaki et al. clarified the effect of improving the TCP performance in sparse networks [13]. However, the reason for the effect is not understood sufficiently. In this paper, we clarify the reason through the discussion using the derived probability distributions of the TCP performance.

The organization of this paper is as follows; Firstly, we discuss related works in Section 2. Then, we introduce an analytic model used in the analysis in Section 3. Further, we derive the probability distributions of the throughput, packet loss probability, and round-trip time of TCP flows in Section 4. Moreover, we investigate the impact of the scale-free structure on the TCP performance, and discuss the validity of our analysis. Finally, we describe the conclusion and future work in Section 7.

2. Related Work

The impact of the scale-free structure on packet transport performance of networks has been investigated in Refs. [14], [15], [16], [17], [18], [19], [20], [21], [22]. In particular, Zhao et al. clarified the impact of the scale-free structure on the packet transmission capacity, which is the maximum number of deliverable packets by routers without the occurrence of network congestion through a numerical simulation experiment [22]. These papers focus on performance of a network layer, but we focus on the end-to-end performance of a transport layer, which is important for network users.

In Refs. [13], [23], the impact of the scale-free structure on the end-end performance of TCP congestion control mechanisms has been investigated through a simulation experiment. In particular, Ohsaki et al. clarified the effect of improving the TCP performance in sparse networks [13]. However, the reason for the effect is not understood sufficiently. In this paper, we analyze the impact of the scale-free structure on the TCP performance, and clarify the reason for the effect on the basis of a fluid-based analysis.

In Refs. [24], [25], the probability distributions of shortest path lengths among nodes and link betweennesses in a scale-free tree are derived, respectively. In Ref. [11], we proposed an analytic method to derive the probability distributions of the TCP performance on the basis of fluid-based analysis utilizing the probability distributions derived in Refs. [24], [25]. In this paper, we discuss the impact of the scale-free structure on the TCP performance using the analytic method proposed in the conference paper [11]. In this paper, we clarify the impact by applying the analytic method proposed in Ref. [11]. In the conference paper [26], we presented the short version of this paper. In this paper, we add the explanation of our analysis in detail, and the numerical results for the validation of our analysis. Therefore, this paper is the extension of the conference papers [11], [26].

Table 1 Definitions of symbols (constants and variables) used in the analytic model.

	network
N	number of nodes
L	set of links
F	set of flows
k_f	flow density
τ	link propagation delay
α	parameter of scale-free tree
	TCP flow i
$s_i(t)$	sending rate
$p_i(t)$	packet loss probability
$d_i(t)$	round-trip time
s_i^*	sending rate at steady state
r_i^*	throughput at steady state
p_i^*	packet loss probability at steady state
d_i^*	round-trip time at steady state
R_i	set of links in the route
h_i	path length (shortest path length)
	link l
$q_l(t)$	current queue length
$p_l(t)$	packet loss probability
$A_l(t)$	arrival rate
C_l	bandwidth
q_l^*	queue length at steady state
p_l^*	packet loss probability at steady state
b_l	betweenness
F_l	set of TCP flows

3. Analytic Model

We describe the definitions of symbol (constants and variables) used in the analytic model in **Table 1**.

3.1 Network

In Ref. [10], Dorogovtsev et al. proposed a network model to be able to generate a tree whose strength of the scale-free property can be adjusted by a parameter. We call this tree *scale-free tree*. In order to analyze the impact of the scale-free structure, we generate scale-free trees with different power indexes of the degree distribution by using the network model [10], and compare the analytic results for the generated scale-free trees.

The degree distribution of a scale-free tree follows the power law with power index $-(2 + \alpha)$ where $\alpha > 0$ [10]. The strength of the scale-free property is defined by the power index of a degree distribution. Hence, we can adjust the power index of the degree distribution in a scale-free tree by parameter α.

Especially if $\alpha = 1$, the power index is -3 which is the same power index of scale-free networks generated by the BA (Barabási–Albert) model [27]. In addition, at the limit of $\alpha \to \infty$, the degree distribution of a scale-free tree approaches Poisson distribution. Hence, if α is set to an extremely large value, we can obtain a scale-free tree with almost no scale-free property. **Figure 2** (a) and (b) show examples of scale-free trees with $\alpha = 1$ and 10, respectively. As compared to the scale-free tree with $\alpha = 10$, that with $\alpha = 1$ has large degree nodes around its root, so degrees of each node when using $\alpha = 1$ largely vary.

3.2 TCP Flow

A source node controls its sending rate by adjusting the window size that is the number of packets available for simultaneous transmission, according to network congestion level and packet processing speed at its destination node. The control by the network congestion level is called as congestion control, and the control by the packet processing speed is called as flow control. We assume the situation where only congestion control affects a sending rate since this paper focuses on the relationship between the TCP performance and the scale-free structure.

In Ref. [28], the dynamics of the sending rate by the TCP congestion control has been modeled with fluid-flow approximation. The fluid-flow approximation is a modeling technique to obtain rough behavior of a TCP flow by averaging the behavior of a TCP congestion control mechanism designed with packet level specification [29], and is used by a large number of studies (see, e.g., Refs. [2], [17], [24], [28]). According to the TCP flow model [28], the dynamics of sending rate $s_i(t)$ of TCP flow i is given by

$$\frac{ds_i(t)}{dt} = \frac{s_i(t - d_i(t))}{s_i(t)d_i(t)^2}(1 - p_i(t)) - \frac{2}{3}s_i(t)s_i(t - d_i(t))p_i(t). \quad (1)$$

where $d_i(t)$ and $p_i(t)$ are the round-trip time and the packet loss probability of TCP flow i, respectively. Note that $y(t)$, R, and $q(t)$ in Eq. (2) of [28] correspond to $s_i(t)$, $d_i(t)$, and $p_i(t)$ in Eq. (1), respectively. In Ref. [28], Ohsaki et al. also modeled the dynamics including the window size reduction by the TCP timeout mechanism. However, we use Eq. (1), which is the equation in the case where the TCP timeout occurrence probability is 0. Namely, to obtain Eq. (1), we substituted TCP timeout probability $p_{\mathrm{TO}}(t) = 0$ into Eq. (2) of Ref. [28]. We assume the situation where congestion control works normally, so the packet loss probability does not become extremely large enough to ingenerate TCP timeout. This assumption is valid for large-scale and wide-area networks like actual networks. That is, for such a network, ignoring the TCP timeout does not affect the validity of the contribution obtained in this paper.

TCP flows following the dynamics given by Eq. (1) are randomly generated in a scale-free tree. Let N_f be the number of TCP flows in a scale-free tree, and the flow density is simply defined by $k_f = N_f/N^2$ because the number of possible source-destination node pairs of TCP flows increases with $O(N^2)$ as N increases. In addition, we select the source-destination node pair of each TCP flow randomly with the equal probability $1/(N(N-1))$, and give the path of each TCP flow by the shortest path between the source/destination nodes. Hence, path length h_i of TCP flow i is identical to its shortest path length between source/destination nodes.

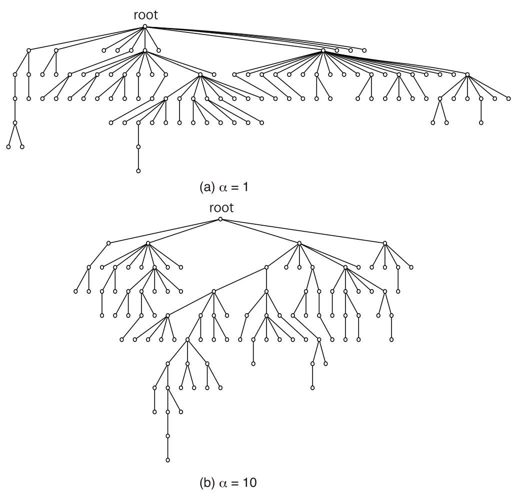

Fig. 2 Examples of scale-free tree ($N = 100$).

3.3 Link Queue

In [30], Liu et al. derived a fluid-flow approximation model of a link queue, and the dynamics of current queue length $q_l(t)$ at link l's input buffer is given by

$$\frac{dq_l(t)}{dt} = \begin{cases} A_l(t) - C_l & \text{if } q_l(t) > 0 \\ (A_l(t) - C_l)^+ & \text{otherwise} \end{cases}, \quad (2)$$

where $(x)^+ = \max(0, x)$, and C_l is the bandwidth of link l. In addition, $A_l(t)$ is the arrival rate of link l's input buffer, and is given by

$$A_l(t) = \sum_{i \in F_l} s_i(t), \quad (3)$$

where F_l is the set of TCP flows passing through link l.

4. Analysis

4.1 TCP Performance at Steady State

We investigate the statistical characteristic of the TCP performance on the scale-free structure. In this investigation, we focus on TCP flows passing through a bottleneck link in a scale-free tree, and derive the probability distribution of the TCP performance by using the analytic method [11]. The objective of our analysis is to understand the qualitative property of the scale-free structure on the TCP performance. The quantitative analysis of the scale-free structure is an issue in the future.

We illustrate the conceptual diagram of a bottleneck link in a scale-free tree in **Fig. 3**. We define a bottleneck link as a link where its bandwidth is fully utilized. We select a bottleneck link l in a scale free tree, and assume betweenness b_l of link l approximately follows the probability distribution

$$P_B(b_l) = \mathrm{Prob}\,(b_l < B \leq b_l + db_l) \simeq -\frac{d\,B_C(b_l)}{d\,b_l}, \quad (4)$$

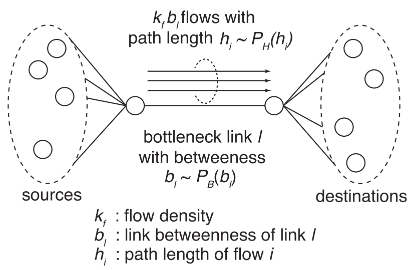

Fig. 3 Conceptual diagram of a bottleneck link in a scale-free tree.

derived in Ref. [24] where we approximate link betweenness b_l as a continuous variable, and $B_C(b_l)$ is given by

$$B_C(b_l) = \frac{(N-\beta)(1-\beta)(N-2\,n_b(b_l)-1)}{(N-1)(n_b(b_l)+1-\beta)(N-n_b(b)-\beta)},$$
$$n_b(b_l) = \frac{N-2}{2} - \frac{N}{2}\sqrt{1-\frac{4\,b_l}{N^2}},$$

where $\beta = 1/(1+\alpha)$. Note that link betweenness b_l is the number of shortest paths passing through link l. We will confirm the validity of this assumption for a qualitative evaluation through a comparison of an optimization-based analysis that considers all links in a scale-free network. Since the source-destination node pair of a TCP flow is also randomly selected with equal probability $1/(N(N-1))$, the expected number of TCP flows passing through bottleneck link l is given by $k_f\,b_l$, and path length h_i of TCP flow i in $k_f\,b_l$ flows approximately follows the probability distribution

$$P_H(h_i) = \text{Prob}\,(h_i < H \le h_i + dh_i) \simeq \frac{e^{-k_h/2}}{\sqrt{\pi\,k_h}}\left(\frac{k_h\,e}{h_i-1}\right)^{h_i-1}, \quad (5)$$

derived on the basis of Ref. [25] (see Appendix). k_h is obtained from

$$e^{k_h/2-1}\sqrt{\pi\,k_h} = \frac{N}{\alpha^{\frac{\alpha}{1+\alpha}}(N(1+\alpha)-1)^{\frac{1}{1+\alpha}}-\alpha}, \quad (6)$$

where we approximate path length h_i as a continuous variable (also see Appendix).

According to the condition $dq_l(t)/dt = 0$ at a steady state, bottleneck link l ($q_l(t) > 0$) satisfies

$$\sum_{i\in F_l} s_i^* = C_l, \quad (7)$$

where s_i^* is the sending rate of TCP flow i at a steady state.

Since $ds_i(t)/dt = 0$ and $s_i(t) = s_i(t-d_i(t))$ at a steady state, s_i^* is given by

$$s_i^* = \frac{1}{d_i^*}\sqrt{\frac{3\,(1-p_i^*)}{2\,p_i^*}}. \quad (8)$$

Let d_i^* and p_i^* be the round-trip time and the packet loss probability of TCP flow i at steady state, respectively. We assume bottleneck link l strongly affects the TCP performance of its passing TCP flows, so ignoring the impact of links other than bottleneck link l for the sake of ease, then d_i^* and p_i^* are given by

$$p_i^* = p_l^*, \quad (9)$$
$$d_i^* = 2\,\tau h_i + \frac{q_l^*}{C_l}, \quad (10)$$

where τ is the propagation delay of a link, and q_l^* is the queue length of bottleneck link l at a steady state. By substituting Eqs. (8) through (10) into Eq. (7), we obtain

$$\sum_{i\in F_l}\frac{1}{d_i^*}\sqrt{\frac{3\,(1-p_i^*)}{2\,p_i^*}} = C_l \quad (11)$$
$$\sqrt{\frac{3\,(1-p_l^*)}{2\,p_l^*}}\sum_{i\in F_l}\frac{1}{2\,\tau\,h_i+\frac{q_l^*}{C_l}} \simeq C_l. \quad (12)$$

If $k_f\,b_l$, which is the number of TCP flows in bottleneck link l, is sufficiently large, Eq. (12) is approximated by

$$\sqrt{\frac{3(1-p_l^*)}{2p_l^*}}k_f\,b_l\,\mu_{d^{-1}} \simeq C_l, \quad (13)$$

where $\mu_{d^{-1}}$ is the mean of $\left(2\,\tau\,h_i + q_l^*/C_l\right)^{-1}$.

From Eqs. (9) and (13), packet loss probability p_i^* of TCP flow i is approximated by

$$p_i^* = p_l^* \simeq \frac{3\,\mu_{d^{-1}}^2}{3\,\mu_{d^{-1}}^2 + 2\,\rho_l^2}, \quad (14)$$

where ρ_i is the link bandwidth per flow, and is given by

$$\rho_l = \frac{C_l}{k_f\,b_l}. \quad (15)$$

By substituting Eqs. (10) and (14) into Eq. (8), sending rate s_i^* of TCP flow i is approximated by

$$s_i^* \simeq \frac{1}{\left(2\,\tau\,h_i + \frac{q_l^*}{C_l}\right)\mu_{d^{-1}}}\rho_l. \quad (16)$$

We denote the throughput of TCP flow i by r_i^*, and approximate r_i^* by $(1-p_i^*)s_i^*$. By substituting Eqs. (14) and (16) into $(1-p_i^*)s_i^*$, throughput r_i^* of TCP flow i is given by

$$r_i^* \simeq \frac{2\,\rho_l^3}{\left(3\,\mu_{d^{-1}}^2 + 2\,\rho_l^2\right)\left(2\,\tau\,h_i + \frac{q_l^*}{C_l}\right)\mu_{d^{-1}}}. \quad (17)$$

4.2 Derivation of Probability Distributions for the TCP Performance

Using a similar method in Ref. [11], we derive the probability distributions of the TCP performance: throughput r_i^*, packet loss rate p_i^*, and round-trip time d_i^*.

Let $P_X(x)$ and $P_{XY}(x,y)$ be the probability distributions (probability densities) of variable x and the simultaneous distribution of variables x and y, respectively. These are defined by

$$P_X(x) := \text{Prob}\,(x < X \le x + dx), \quad (18)$$
$$P_{XY}(x,y) := \text{Prob}\,(x < X \le x + dx, y < Y \le y + dy). \quad (19)$$

$P_X(x)\,dx$ and $P_{XY}(x,y)\,dx\,dy$ represent the probability of x and the probability of (x,y), respectively.

If x is equal to injective and differentiable function $f(y)$, the probability of x is equivalent to that of y. Namely,

$$P_X(x)\,dx = P_Y(y)\,dy \quad (20)$$
$$P_X(x) = P_Y(y)\frac{dy}{dx} = P_Y(f^{-1}(x))\frac{df^{-1}(x)}{dx}. \quad (21)$$

Equation (21) means the change of variables between probability distributions $P_X(x)$ and $P_Y(y)$. Using such a change of variables, we can derive $P_X(x)$ from $f(y)$ and $P_Y(y)$. With the same way, if x is equal to injective and differentiable function $g(y,z)$, the probability of (x,y) is equivalent to that of (y,z). Namely,

$$P_{XY}(x,y)\,dx\,dy = P_{YZ}(y,z)\,dy\,dz \quad (22)$$
$$P_{XY}(x,y) = P_{YZ}(y,z)\frac{dz}{dx} = P_{YZ}(y, g_z^{-1}(x,y))\frac{dg_z^{-1}(x,y)}{dx}, \quad (23)$$

where $g_z^{-1}(x, y)$ is the inverse function of $g(y, z)$ for z. Equation (24) means the change of variables between probability distributions $P_{XY}(x, y)$ and $P_{YZ}(y, z)$. Since the probability of x is given by the total probability of (x, y) for any y, $P_X(x)$ is derived as

$$P_X(x) = \int P_{XY}(x, y)\, dy = \int P_{YZ}(y, g_z^{-1}(x, y))\, \frac{dg_z^{-1}(x, y)}{dx}\, dy. \quad (24)$$

To derive the probability distributions of the TCP performance, we utilize the following results:

- probability distributions $P_B(b_l)$ and $P_H(h_i)$ of link betweenness b_l and path length h_i, derived in Refs. [24], [25],
- Equations (10), (14), (15), and (17), which means the relations among link betweenness b_l, path length h_i, and the TCP performance.

Using Eq. (24), $P_R(r_i^*)$ is derived as

$$P_R(r_i^*) = \int P_{BH}(b_l, u_{h_i}^{-1}(r_i^*, b_l))\frac{du_{h_i}^{-1}(r_i^*, b_l)}{dr_i^*}db_l = \int P_B(b_l)\, P_H(u_{h_i}^{-1}(r_i^*, b_l))\frac{du_{h_i}^{-1}(r_i^*, b_l)}{dr_i^*}db_l, \quad (25)$$

where $u_{h_i}^{-1}(r_i^*, b_l)$ is the inverse function of Eqs. (15) and (17) for h_i. Note that $P_{BH}(b_l, h_i) = P_B(b_i)\, P_H(h_i)$ since b_l and h_i are independent mutually. Using Eq. (21), $P_P(p_i^*)$ is derived as

$$P_P(p_i^*) = P_B(v_{b_l}^{-1}(p_i^*))\frac{dv_{b_l}^{-1}(p_i^*)}{dp_i^*}, \quad (26)$$

where $v_{b_l}^{-1}(p_i^*)$ is the inverse function of Eqs. (14) and (15) for b_l. In addition, $P_D(d_i^*)$ is derived as

$$P_D(d_i^*) = P_H(w_{h_i}^{-1}(d_i^*))\frac{dw_{h_i}^{-1}(d_i^*)}{dd_i^*}, \quad (27)$$

where $w_{h_i}^{-1}(d_i^*)$ is the inverse function of Eq. (10) for h_i.

5. Numerical Example

We investigate the impact of the scale-free structure on the TCP performance using the probability distributions derived in Section 4. Firstly, we calculate the means of throughput r_i^*, packet loss probability p_i^*, and round-trip time d_i^* of TCP flow i passing through bottleneck link l using the probability distributions given by Eqs. (25) though (27). Then, we check up on the impact of α, which indicates the strength of the scale-free property, on these means.

Moreover, we confirm the validity of our analysis through a comparison with an optimization-based analysis [12]. In Ref. [12], Low et al. describe an optimization framework to analyze the performance of congestion control mechanisms at a steady state. Low's framework is widely used in a large number of papers (see, e.g., Refs. [31], [32], [33]), so its effectiveness is confirmed well.

Unless otherwise noted, we use the parameter setting shown in **Table 2**.

Table 2 Default parameter setting used in the numerical example.

common			
number of nodes	N	1,000	
link bandwidth	C_l	1,000	[Mbit/s]
link propagation delay	τ	0.001	[s]
our analysis			
queue length at steady state in our analysis	q_l^*	1.25×10^6	[byte]
flow density	k_f	0.1	
Low's framework			
parameter of the gradient method	γ	0.1	
repeat count	t_E	1000	
total queue length in routes	q_r^*	25.0×10^6	[byte]

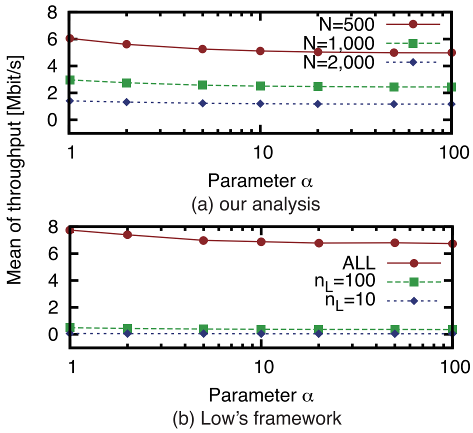

Fig. 4 Mean of throughput r_i^* as α changes when using different N's.

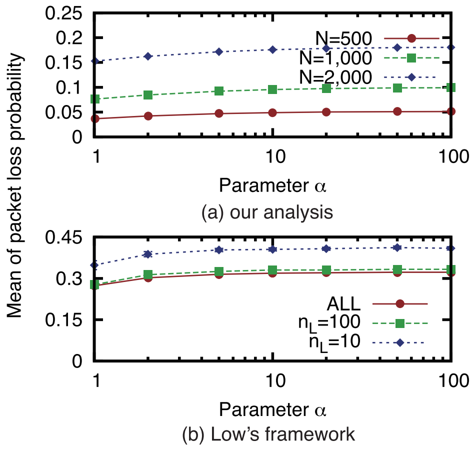

Fig. 5 Mean of packet loss probabilities p_i^* as α changes when using different N's.

5.1 Impact of Scale-Free Structure on the TCP Performance

Figures 4 (a) through **6** (a) show the means of throughput r_i^*, packet loss probability p_i^*, and round-trip time d_i^* calculated by Eqs. (25) through (27) as α changes when using different N's, respectively [*1,*2]. When looking at Fig. 4 (a), the mean of throughput r_i^* increases as α decreases. In addition, when looking at **Figs. 5** (a) and 6 (a), the means of packet loss probability p_i^* and round-trip time d_i^* decrease as α decreases. From the sum of these results, it can be said that the scale-free structure acts in the di-

*1 In Fig. 1 (a) of Ref. [26], we have shown sending rate s_i^* instead of throughput r_i^*. Since r_i^* should be more important performance than s_i^*, we show r_i^* in Fig. 4 (a) of this paper.

*2 The mean of throughput r_i^* shown in Fig. 4 (a) looks identical to that of sending rate s_i^* shown in Fig. 1 (a) of Ref. [26]. This is explained by the following reason. If packet loss probability p_i^* is small, both s_i^* and r_i^* are large, and s_i^* is almost same with r_i^*. Since mean is strongly affected by samples with large value, the mean of s_i^* is also almost same with that of r_i^*.

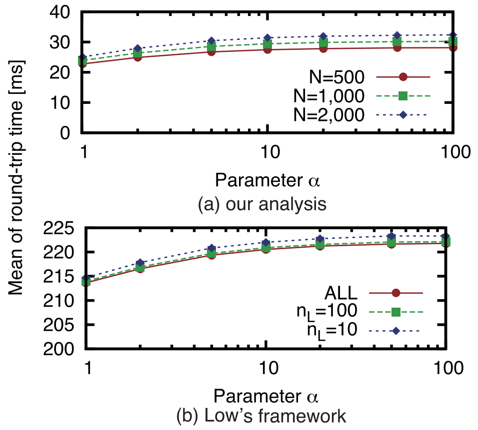

Fig. 6 Mean of round-trip time d_i^* as α changes when using different N's.

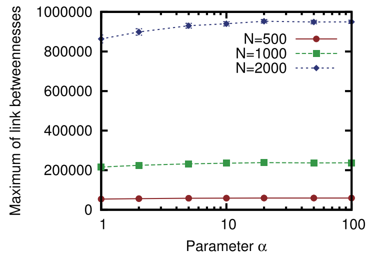

Fig. 7 Maximum value of link betweenness in a scale-free tree as α changes when using different N.

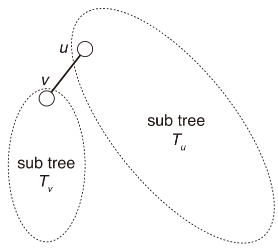

Fig. 8 Conceptual diagram of sub trees T_u and T_v in case of dividing a scale-free tree by link (u, v).

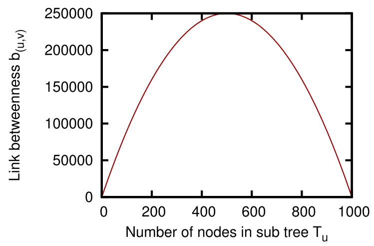

Fig. 9 Link betweenness given by Eq. (28) (N = 1,000).

rection of improving the TCP performance. In addition, this trend also matches the reported simulation result for sparse networks in Ref. [13]. In our analysis, we consider the heterogeneity of TCP flows in term of shortest path length unlike the Fekete's analysis [24]. We describe the difference between results of the homogeneous case and the heterogeneous case (Figs. 4 (a) through 6 (a)) as follows. In the homogeneous case, as α decreases, shortest path lengths are same for different α's. Hence, the mean of round-trip time d_i^* does not change unlike Fig. 6 (a). Packet loss probability p_i^* does not depend on shortest path length h_i. Hence, we also obtain the same results shown in Fig. 5 (a). Thirdly, in the homogeneous case, as α decreases, the mean of throughput r_i^* increases more gently compared to Fig. 4 (a) since the mean of round-trip time d_i^* is same for different α's.

The reason why the round-trip time is small when setting α to a small value is easy to explain because the scale-free structure has the effect of making the average of shortest path lengths among nodes smaller. On the other hand, the reason why the packet loss probability is small when setting α to a small value is hard to be explained intuitively because traffic of TCP flows concentrates to links connected to a hub node in the scale-free structure. In addition, the reason why the throughput is large when setting α to a small value can be explained by the small packet loss probability and the small round-trip time. To understand the mechanism which brought the large throughput when setting α to a small value, we should clarify the reason of packet loss probability getting small in the scale-free structure.

In order to clarify the reason for the small packet loss probability in the scale-free structure, we confirm the maximum of link betweennesses in scale-free trees. This is because that the mean of loss probabilities in the scale-free structure is largely influenced by betweennesses of links connected to a hub node, which largely generates packet losses. **Figure 7** shows the maximum of link betweennesses in scale-free trees as α changes when using different N. When looking at Fig. 7, the maximum of link betweennesses decreases as α decreases. Note that we discuss the maximum values in Fig. 7 unlike Figs. 4 through 6. According to Fig. 7, the reason of packet loss probability getting small can be explained by the maximum of link betweennesses getting small. However, the maximum of link betweennesses when setting α to a small value should be large because many shortest paths intuitively are included in links connected to a hub node in the scale-free structure.

Then, we discuss the reason of the scale-free structure making the maximum of link betweennesses to be small. Now, we consider betweenness $b_{(u,v)}$ of the link with nodes u and v in a scale-free tree. Let T_u and T_v be sub trees including node u and node v as a root node, respectively. **Figure 8** shows conceptual diagram of sub trees T_u and T_v. Since we use a tree as a network topology, link betweenness $b_{(u,v)}$ is given by

$$b_{(u\,v)} = N_u\,N_v = N_u\,(N - N_u), \tag{28}$$

where N_u and N_v are the numbers of nodes in sub trees T_u and T_v, respectively. **Figure 9** shows the link betweenness given by Eq. (28) if N = 1,000.

According to Eq. (28), link betweenness $b_{(u\,v)}$ becomes maximum at $N_u = N/2$. This indicates that if there is a link where a tree can be divided more equally, the maximum link betweenness of the tree becomes larger. Such a link exists hardly if there is a large degree node in the vicinity of the root. This is because when we add/delete the large degree node to/from T_u, N_u largely increases/decreases beyond $N/2$. **Figure 10** shows examples of dividing a tree with/without a large degree node in the vicinity of its root (N = 10). In this figure, we write the combination of N_u and N_v when dividing a tree at each link. From these examples, one can intuitively find that it is difficult to divide a tree if there is a large degree node in the vicinity of its root. According to the network model of scale free tree [10], as the scale-free property

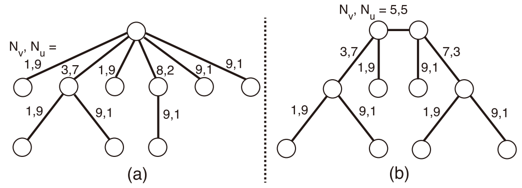

Fig. 10 Examples of dividing a tree with/without a large degree node in the vicinity of its root ($N = 10$).

becomes strong, a large degree node in the vicinity of the root exists more easily. Therefore, the strength of the scale-free property decreases the maximum link betweenness.

In summary of the above discussion, the scale-free structure has the effect of making link betweenness smaller, and it can be said that packet loss probability gets smaller when α is set to a small value as the result. Although we slightly digress from the main topic, according to the above discussion, the star topology is optimal for the TCP performance because it has the smallest average of path length and the smallest link betweenness.

5.2 Validation of Our Analysis

We confirm the validity of our analysis through a comparison with the optimization-based analysis based on Low's framework. In Section 4.1, we focused on a bottleneck link for the sake of ease, whereas there are many links in a scale-free tree. Hence, in this section, we confirm the validity of such a simplification. Fortunately, Low's framework consider all links in a network. Hence, comparing our analysis results with the results of Low's framework, we can confirm the validity of such a simplification. We first obtain numerical results on the basis of Low's framework, and then compare our numerical results with those of Low's framework for the validation.

In Low's framework, congestion control is formulated as the optimization problem of sending rates $\boldsymbol{s}^* = (s_1^*, ..., s_{|F|}^*)$ with the following objective function

$$\max_{\boldsymbol{s}^*} \sum_{i \in F} U_i(s_i^*) \tag{29}$$
$$\text{s.t.} \quad \sum_{i \in F_l} s_i^* \le C \quad \forall l \in L$$
$$s_i^* > 0 \quad \forall i \in F$$

where $U_i(s_i^*)$ is called as *utility function*, which is defined by

$$U_i(s_i^*) = \int p_i^*(s_i)\, ds_i, \tag{30}$$

with boundary condition $p_i^*(0) = 0$. $p_i^*(s_i)$ is the inverse function of Eq. (8) for packet loss probability p_i^*. By substituting Eq. (8) into Eq. (30), utility function $U_i(s_i^*)$ corresponding to our analytic model is given by

$$U_i(s_i^*) = \frac{1}{d_i^*} \sqrt{\frac{3}{2}} \tan^{-1}\left(\sqrt{\frac{3}{2}}\, d_i^*\, s_i^* \right). \tag{31}$$

According to Eq. (31), $\partial^2 U_i(s_i^*)/\partial s_i^2$ is always negative in $[0, \infty]$. Hence, the objective function is concave, so Eq. (29) can be numerically solved by the gradient method of non-linear programming [34].

Whereas we focus on the effect of bottleneck links on the TCP performance in analysis, Low's framework considers the effect of not only bottleneck links but also non-bottleneck links. Hence, Low's framework can capture the complex feature of large-scale networks, but the essential feature of large-scale networks would be buried in the complex feature.

Using the gradient method, throughput r_i^*, packet loss probability p_i^*, and round-trip time d_i^* at a steady state are obtained from the following procedures.

(1) Set h_i and calculate $d_i^* = 2\,\tau\, h_i + q_r^*/C_l$ from a given network. q_r^* is the total queue length in routes of TCP flows.

(2) Initialize packet loss probability $p_l(0)$ of all links l, and sending rates $s_i(0)$ of all TCP flows.

(3) Repeat the following procedures from $t = 1$ to $t = t_E$.

(a) Calculate arrival rate $A_l(t)$ of link l by the following equation

$$A_l(t) = \sum_{i \in F_l} s_i(t). \tag{32}$$

(b) Calculate packet loss probability $p_l(t)$ of link l by the following equation

$$p_l(t) = \Big[p_l(t-1) - \gamma\, p_l(t-1) \big(A_l(t) - C_l \big) \Big]^+, \tag{33}$$

where γ is a positive coefficient. As we update $p_l(t)$ using Eq. (33), $p_l(t)$ converges to an adequate steady solution p_l^*. According to Eq. (33), $p_l(t)$ continues to change until $p_l(t-1)(A_l(t) - C_l) = 0$. For a non bottleneck link l, we obtain $p_l^* = 0$ since $A_l(t) < C_l$ and $\lim_{t \to \infty} p_l(t-1)(A_l(t) - C_l) = 0$. For a bottleneck link l, we obtain $p_l^* \ge 0$ since $A_l(t) = C_l$ and $\lim_{t \to \infty} p_l(t-1)(A_l(t) - C_l) = 0$.

(c) Calculate packet loss probability $p_i(t)$ of TCP flow i by the following equation

$$p_i(t) = \sum_{l \in R_i} p_l(t), \tag{34}$$

where R_i is the set of links in the route of TCP flow i. Note that the right-hand side of the above equation is the approximation of $1 - \prod_{l \in R_i}(1 - p_l(t))$.

(d) Calculate sending rate $s_i(t)$ of TCP flow i by Eq. (8).

(e) Increment t.

(4) Obtain throughput r_i^* and packet loss probability p_i^* of TCP flow i from

$$r_i^* = (1 - p_i(t_E))\, s_i(t_E), \tag{35}$$
$$p_i^* = p_i(t_E). \tag{36}$$

Figures 4 (b) through 6 (b) show the mean of throughput r_i^*, packet loss probability p_i^*, and round-trip time d_i^* obtained from Low's framework. In these results, we show the mean of the TCP performance of flows passing through all links and n_L links. To obtain the results for $n_L = 10$ and 100, we select n_L links in descending order of link betweenness, take the TCP performance of flows passing through the selected links from the results with all links, and calculate the mean of the TCP performance for the flows. From these results, we observe the same qualitative property found in our numerical results. That is, the trend of the TCP

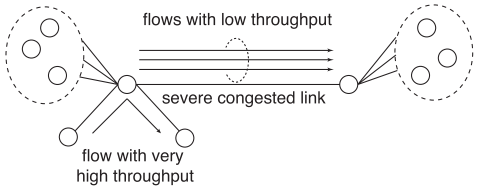

Fig. 11 Multiple bottleneck links in the Low's framework model.

performance in our analysis when α changes is the same in that of Low's framework. For instance, as α decreases, both throughputs of our analysis and Low's framework increase. However, their values are large different from a quantitative perspective. This is caused by a simple reason. Whereas we focus on the effect of bottleneck links on the TCP performance in analysis, Low's framework considers the effect of not only bottleneck links but also non-bottleneck links. Because of this, packet loss probabilities and round-trip times obtained from Low's framework are larger than those obtained from our analysis results. According to Fig. 4, the throughput obtained from Low's framework is also larger than that obtained from our analysis results. This is caused by the following reason. Since Low's framework considers many links, there is a TCP flow with a very high throughput in Low's framework. Such a TCP flow uses the rest of link bandwidth used by flows passing through a severely congested link (see **Fig. 11**), so the mean of the throughput in Low's framework is larger than that in our analysis although the mean of the packet loss probability in Low's framework is larger than that in our analysis.

According to the comparison, we conclude that our analysis is valid for qualitative evaluation.

6. Discussion

Finally, we discuss the superiority of our analysis compared with Low's framework. The superiority of our analysis is summarized as follow.

- To be able to analytically derive the probability distributions of the TCP performance
- To be able to clarify the reason why the scale-free structure improves the TCP performance in sparse networks, previously shown in Ref. [13]

Since the Low's framework model includes many links, it is considered impossible to analytically derive the probability distributions of the TCP performance due to the complexity of the model. In this paper, we focused on a bottleneck link because it would strongly affect the qualitative property of the TCP performance, and succeeded in the derivation of the probability distributions for using qualitative evaluation.

In Section 4.1, we showed the same effect of the scale-free structure shown in Ref. [13] through our analysis focusing on a bottleneck link in a scale-free structure. This indicates that we can narrow down the cause of the effect to the bottleneck link. In the result, we succeeded in the clarification of the reason why the scale-free structure improves the TCP performance through the discussion of traffic intensity (link betweenness) at the bottleneck link.

7. Conclusion and Future Work

In this paper, we clarified the impact of the scale-free structure on the TCP performance using the analysis based on the fluid-based analysis. We investigated statistical characteristic of the TCP performance in the scale-free structure. In the investigation, we focused on TCP flows passing through a bottleneck link in a scale-free tree, and derived the probability distribution for the TCP performance of TCP flows by using the analytic method [11]. Using several numerical examples, we clarified that the scale-free structure improves the TCP performance because of a reduction in the average path length and also a reduction of the traffic intensity at the bottleneck link in the scale-free structure. Furthermore, we confirmed the validity of our analysis through a comparison with the optimization-based analysis by Low et al. According to the comparison, we conclude that our analysis is valid for qualitative evaluation.

As future work, we are planning to analyze the impact of the scale-free structure on the TCP performance when network topology is not a tree. Then, we will clarify the optimal congestion control in the scale-free structure. Moreover, we will develop a method to analyze the quantitative property of the scale-free structure on the TCP performance.

Acknowledgments This work was supported by JSPS KAKENHI Grant Numbers 25280030 and 15K15985.

References

[1] Faloutsos, M., Faloutsos, P. and Faloutsos, C.: On power-law relationships of the internet topology, *ACM SIGCOMM Computer Communication Review*, Vol.29, No.4, pp.251–262 (1999).

[2] Misra, V., Gong, W.-B. and Towsley, D.: Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED, *ACM SIGCOMM Computer Communication Review*, Vol.30, No.4, pp.151–160 (2000).

[3] Ohsaki, H. and Murata, M.: Steady state analysis of the RED gateway: Stability, transient behavior, and parameter setting, *IEICE Trans. Commun.*, Vol.85, No.1, pp.107–115 (2002).

[4] Grieco, L.A. and Mascolo, S.: Performance evaluation and comparison of Westwood+, New Reno, and Vegas TCP congestion control, *ACM SIGCOMM Computer Communication Review*, Vol.34, No.2, pp.25–38 (2004).

[5] Barakat, C., Altman, E. and Dabbous, W.: On TCP performance in a heterogeneous network: A survey, *IEEE Communications Magazine*, Vol.38, No.1, pp.40–46 (2000).

[6] Foong, A.P., Huff, T.R., Hum, H.H., Patwardhan, J.P. and Regnier, G.J.: TCP performance re-visited, *Proc. IEEE International Symposium on Performance Analysis of Systems and Software* (*ISPASS 2003*), pp.70–79 (2003).

[7] Chen, F., Chen, Z., Wang, X. and Yuan, Z.: The average path length of scale free networks, *Communications in Nonlinear Science and Numerical Simulation*, Vol.13, No.7, pp.1405–1410 (2006).

[8] Padhye, J., Firoiu, V., Towsley, D. and Kurose, J.: Modeling TCP throughput: A simple model and its empirical validation, *ACM SIGCOMM Computer Communication Review*, Vol.28, No.4, pp.303–314 (1998).

[9] Kim, D.-H., Noh, J.D. and Jeong, H.: Scale-free trees: The skeletons of complex networks, *Physical Review E*, Vol.70, No.4, p.046126 (2004).

[10] Dorogovtsev, S.N., Mendes, J.F.F. and Samukhin, A.N.: Structure of growing networks with preferential linking, *Physical Review Letters*, Vol.85, No.21, p.4633 (2000).

[11] Sakumoto, Y., Ohsaki, H. and Imase, M.: Fluid-based analysis of TCP flows in a scale-free network, *Proc. 11th IEEE/IPSJ International Symposium on Applications and the Internet* (*SAINT 2011*), pp.37–43 (2011).

[12] Low, S.H., Paganini, F. and Doyle, J.C.: Internet congestion control, *IEEE Control Systems*, Vol.22, No.1, pp.28–43 (2002).

[13] Ohsaki, H. and Imase, M.: On the effect of scale-free structure of

network topology on TCP performance, *Proc. IEEE Communications Quality and Reliability Workshop 2008* (*CQR 2008 CD-ROM*) (2008).
[14] Tadić, B. and Rodgers, G.: Packet transport on scale-free networks, *Advances in Complex Systems*, Vol.5, No.4, pp.445–456 (2002).
[15] Ohira, T. and Sawatari, R.: Phase transition in a computer network traffic model, *Physical Review E*, Vol.58, No.1, p.193 (1998).
[16] Valverde, S. and Solé, R.V.: Self-organized critical traffic in parallel computer networks, *Physica A: Statistical Mechanics and its Applications*, Vol.312, No.3, pp.636–648 (2002).
[17] Liu, Z., Ma, W., Zhang, H., Sun, Y. and Hui, P.M.: An efficient approach of controlling traffic congestion in scale-free networks, *Physica A: Statistical Mechanics and its Applications*, Vol.370, No.2, pp.843–853 (2006).
[18] Wang, D., Jing, Y. and Zhang, S.: Traffic dynamics based on a traffic awareness routing strategy on scale-free networks, *Physica A: Statistical Mechanics and its Applications*, Vol.387, No.12, pp.3001–3007 (2008).
[19] Ling, X., Hu, M.-B., Jiang, R. and Wu, Q.-S.: Global dynamic routing for scale-free networks, *Physical Review E*, Vol.81, No.1, p.016113 (2010).
[20] Yang, H.-X., Wang, W.-X., Wu, Z.-X. and Wang, B.-H.: Traffic dynamics in scale-free networks with limited packet-delivering capacity, *Physica A: Statistical Mechanics and its Applications*, Vol.387, No.27, pp.6857–6862 (2008).
[21] Zhang, G.-Q., Wang, D. and Li, G.-J.: Enhancing the transmission efficiency by edge deletion in scale-free networks, *Physical Review E*, Vol.76, No.1, p.017101 (2007).
[22] Zhao, L., Lai, Y.-C., Park, K. and Ye, N.: Onset of traffic congestion in complex networks, *Physical Review E*, Vol.71, No.2, p.026125 (2005).
[23] Ohsaki, H., Yagi, K. and Imase, M.: On the effect of scale-free structure of network topology on end-to-end performance, *Proc. 7th IEEE/IPSJ International Symposium on Applications and the Internet* (*SAINT 2007*), pp.12–18 (2007).
[24] Fekete, A., Vattay, G. and Kocarev, L.: Traffic dynamics in scale-free networks, *Complexus*, Vol.3, No.1-3, pp.97–107 (2006).
[25] Szabó, G., Alava, M. and Kertész, J.: Shortest paths and load scaling in scale-free trees, *Physical Review E*, Vol.66, No.2, p.26101 (2002).
[26] Sakumoto, Y. and Ohsaki, H.: On the Impact of Scale-Free Structure on End-to-End TCP Performance, *Proc. 39th IEEE Computer Society International Conference on Computers, Software & Applications* (*COMPSAC 2015*), Vol.3, pp.652–653, IEEE (2015).
[27] Bárabasi, A.L. and Albert, R.: Emergence of scaling in random networks, *Science*, Vol.286, No.5439, pp.509–512 (1999).
[28] Ohsaki, H., Ujiie, J. and Imase, M.: On scalable modeling of TCP congestion control mechanism for large-scale IP networks, *Proc. 5th IEEE/IPSJ International Symposium on Applications and the Internet* (*SAINT 2005*), pp.361–369 (2005).
[29] Allman, M., Paxson, V. and Blanton, E.: TCP Congestion Control, RFC 5681 (2009).
[30] Liu, Y., Presti, F.L., Misra, V., Towsley, D.F. and Gu, Y.: Scalable fluid models and simulations for large-scale IP networks, *ACM Trans. Modeling and Computer Simulation* (*TOMACS*), Vol.14, No.3, pp.305–324 (2004).
[31] Xiao, L., Johansson, M. and Boyd, S.P.: Simultaneous routing and resource allocation via dual decomposition, *IEEE Trans. Communications*, Vol.52, No.7, pp.1136–1144 (2004).
[32] Wei, D.X., Jin, C., Low, S.H. and Hegde, S.: FAST TCP: Motivation, architecture, algorithms, performance, *IEEE/ACM Trans. Networking* (*ToN*), Vol.14, No.6, pp.1246–1259 (2006).
[33] Carofiglio, G., Gallo, M., Muscariello, L., Papalini, M. and Wang, S.: Optimal multipath congestion control and request forwarding in information-centric networks, *Proc. 21st IEEE International Conference on Network Protocols* (*ICNP 2013*), pp.1–10, IEEE (2013).
[34] Bertsekas, D.P.: *Nonlinear programming*, Athena Scientific (1999).

Appendix

Deriving Probability Distribution of Path length h_i on the basis of Ref. [25]

In Ref. [25], Szabó et al. approximately derived $n(l)$, which is the number of nodes with l distances (hops) far from the root node of a tree. According to Eq. (6) in Ref. [25], $n(l)$ is approximately given by

$$n(l) \approx \frac{b(0)}{e^{\lambda}} \left(\frac{A\,e}{l-1}\right)^{\lambda(l-1)}, \tag{A.1}$$

where A and λ are positive constants. $b(0)$ is the degree of the root node. With the same approximation method, N is approximated by

$$N \approx \frac{b(0)}{e^{\lambda}} e^{A\lambda} \sqrt{\frac{2\pi A}{\lambda}} \tag{A.2}$$

(see Eq. (8) in Ref. [25]).

Using Eqs. (A.1) and (A.2), probability distribution $P_L(l)$ of root-to-node distance l is given by

$$P_L(l) = \frac{n(l)}{N} \approx e^{-A\lambda} \sqrt{\frac{\lambda}{2\pi A}} \left(\frac{A\,e}{l-1}\right)^{\lambda(l-1)}. \tag{A.3}$$

In Ref. [25], using the curve fitting based on Eq. (A.3) with $\lambda = 1$, probability distribution $P_H(h)$ of node-to-node distance (path length) h is experimentally derived as

$$P_H(h) \approx \frac{e^{-A}}{\sqrt{2\pi A}} \left(\frac{2A\,e}{h-1}\right)^{h-1} \tag{A.4}$$

(see Fig. 3 in Ref. [25]). By substituting $k_h = 2A$ and $h_i = h$ to Eq. (A.4), we obtain Eq. (5).

In the network model of scale free trees [10], the root node corresponds to the node that is firstly added in a tree. Hence, the degree of the root node, $b(0)$, is given by

$$b(0) = \alpha^{\frac{\alpha}{1+\alpha}} (N(1+\alpha) - 1)^{\frac{1}{1+\alpha}} - \alpha. \tag{A.5}$$

By substituting Eq. (A.5) to Eq. (A.2) with $\lambda = 1$ and $k_h = 2A$, we obtain Eq. (6).

Yusuke Sakumoto received his M.E. and Ph.D. degrees in the Information and Computer Sciences from Osaka University in 2008 and 2010, respectively. He is currently an assistant professor at Graduate School of System Design, Tokyo Metropolitan University, Japan. His research work is in the area of performance evaluation of computer networks. He is a member of IEEE, IPSJ and IEICE.

Hiroyuki Ohsaki received his M.E. degree in the Information and Computer Sciences from Osaka University, Osaka, Japan, in 1995. He also received his Ph.D. degree from Osaka University, Osaka, Japan, in 1997. He is currently an associate professor at Department of Information Networking, Graduate School of Information Science and Technology, Osaka University, Japan. His research work is in the area of traffic management in high-speed networks. He is a member of IEEE, IEICE, and IPSJ.