On Scalable Modeling of TCP Congestion Control Mechanism for Large-Scale IP Networks

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Abstract

In this paper, we propose an analytic approach of modeling a closed-loop network with multiple feedback loops using fluid-flow approximation. Specifically, we model building blocks of a network (i.e., the congestion control mechanism of TCP, propagation delay of a transmission link, and the buffer of a router) as independent continuous-time systems. By interconnecting these systems, we obtain the model for a complex closed-loop network. We improve the accuracy of analytic models for TCP congestion control and RED router by extending existing fluid-flow models. First, we obtain a block diagram for each continuous-time system using a standard CAD tool widely used in control engineering. Second, we evaluate the performance of a closed-loop network with multiple feedback loops by connecting these block diagrams. We also validate the effectiveness of our analytic approach by comparing our analytic results with simulation results. Unlike other fluid-based modeling approaches, our analytic approach is scalable *and* accurate*; our analytic approach is scalable in terms of the number of TCP connections and routers since both input/output of all continuous-time systems are uniformly defined as a packet transmission rate. Our analytic approach is accurate since the timeout mechanism of TCP and the packet dropping algorithm of RED router are rigorously modeled in our continuous-time systems.*

1 Introduction

In recent years, the number of nodes connected to the Internet and the number of users using the Internet have increased with exponential momentum. Along with this, demands with respect to design techniques and performance analysis techniques for large-scale networks have been rising. However, the current situation is one where performance analysis techniques for large-scale networks are not adequately provided.

In the past, a variety of research has been performed with regard to performance evaluation techniques for networks using mathematical analysis techniques. However, the networks in question are mostly small-scale. In addition, a variety of research has been performed with regard to simulation techniques as performance evaluation techniques for large-scale networks as well. However, simulation techniques are also mostly intended for small-scale networks.

Thus, an analysis technique for multistage-connected closed-loop networks with multiple feedback loops is proposed in the current paper. Here, a feedback loop means feedback control performed by transport-layer communication protocol such as TCP. In the analytic approach proposed in this paper, by extending the existing analytic approach proposed in [7], we model individual components of a network as independent SISO (Single-Input Single-Output) continuous-time systems using fluid-flow approximation.

The TCP congestion control mechanism, link propagation delay, and router buffer are each modeled as components of the network. Then, the entire closed-loop network is modeled via multistage connection of a continuous-time system of individual components. Modeling of large-scale networks is possible via multistage connection of a continuous-time system modeling individual components of a network.

Moreover, performance evaluation is performed with respect to the model of the entire network obtained using a control engineering CAD tool. Specifically, a continuous-time system indicating individual components of a network is first expressed in a block diagram through use of the control engineering CAD tool. Next, a network having multiple feedback loops is expressed via connection of each block diagram and performance analysis is performed via numerical simulation and the like. Moreover, comparison of results of steady state analysis using the analysis technique proposed and simulation results is performed and the validity of approximation analysis is verified.

The current paper's structure is as follows. In Section 2, related research with regard to modeling of networks using a method of fluid approximation and performance evaluation of large-scale networks is first presented. In Section 3, we explain our analytic approach that models building blocks of a network as continuous-time systems. In Section 3, an analysis technique proposed with modeling of individual components of a network via a continuous-time system is explained. In Section 4, we perform steady-state analysis using the analytic models presented in Section 3. In Section 5, the efficacy of the analysis technique proposed is verified via comparison of analytical results and simulation results. Finally, a summary of the current paper and future topics are discussed in Section 6.

2 Related Work

In the past, a variety of research has been performed with regard to modeling of networks using methods of fluid approximation [7, 8]. In these studies, a duality model for the TCP congestion control mechanisms (TCP Reno and TCP Vegas) and network are proposed and the fact that the TCP congestion control mechanism can be viewed as an optimization algorithm that maximizes the utility function is indicated. Moreover, values for equilibrium (maximization of the utility function) derived from the duality model and simulation results are compared and the validity of the duality model is indicated.

In addition, there is research with the validity of the duality model verified via simulation testing [3]. In this paper, the packet loss probability of a RED router derived by simulation and the packet loss probability of a RED router at equilibrium obtained via a duality model are compared. The results indicate that both values are almost identical. However, the RED router's average queue length and TCP window size were not compared. In addition, there was a focus on the value for equilibrium alone in [3] and dynamics prior to equilibrium are not indicated. On the other hand, analysis of dynamics prior to equilibrium is also possible with the analysis technique proposed in the current paper.

In [7, 8, 3], input/output of the model for the TCP congestion control mechanism and input/output of the model for RED router differ. Thus, there is a problem when modeling a large-scale network in that the structure of the model becomes more complex. In contrast, modeling of large-scale networks can readily be performed with the analysis technique proposed in the current paper because input/outputs for components of the network are all standardized to the packet transmission rate.

In addition, numerous research projects to speed up simulation of large-scale networks through use of a fluid approximation model have been conducted [10, 6, 2]. In addition, performance evaluation techniques for large-scale networks using a fluid approximation model have been proposed in the literature [6]. The fluid approximation model proposed in [9] is expanded and the TCP congestion control mechanism and active queue management mechanism are modeled. In addition, the effects of the maximum transmission rate for packets being suppressed by the router's processing capacity are also modeled through explicit modeling of the order in which individual TCP connections pass through the router.

The TCP Reno window size and router queue length are then derived at equilibrium through numerically solving a differential equation by repeated calculation. Repeated calculation of this differential equation is stated to be considered approximating simulation. In this paper, speeding up of numerical calculation is sought through integration of multiple TCP connections passing through the exact same path as one class.

Unlike these fluid-based modeling approaches [7, 8, 3, 6, 9], our analytic approach is *scalable* and *accurate*. Namely, our analytic approach is scalable in terms of the number of TCP connections and routers since both input/output of all continuous-time systems are uniformly defined as a packet transmission rate. Also, our analytic approach is accurate since the timeout mechanism of TCP and the packet dropping algorithm of RED router are rigorously modeled in our continuous-time systems.

3 Modeling Individual Network Components using Fluid Approximation

Our analysis technique is explained in this section. An analytical model proposed in the literature [7] is extended with the analysis technique proposed in the current paper. Specifically, components of a network (TCP congestion control mechanism, router buffer, and link propagation delay) are each modeled as independent continuous-time systems by a method of fluid approximation.

In the current paper, for the sake of simplicity, only TCP Reno is modeled as the TCP congestion control mechanism. Hereafter, TCP Reno is simply denoted as TCP unless noted otherwise. Next is modeling of the router buffer, although in the current paper, for the sake of simplicity, only RED that is an active queue management router is modeled. Finally, the link propagation delay is modeled simply as a delay element of the signal. An entire closed-loop network is modeled with multistage connection of each continuous-time system (Fig. 1). In our analytic approach, since both input and output of each model are unified as the packet transmission rate, we can simply interconnect these continuous-time systems.

Hereafter, the model of the TCP congestion control mechanism, RED router buffer, and link propagation delay are explained. Definitions of symbols (constants and

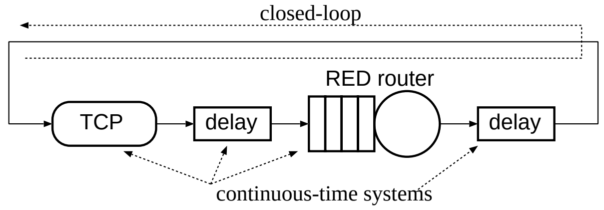

Figure 1. Modeling entire network by interconnecting continuous-time systems

Table 1. Definitions of symbols (constants and variables)

$x(t)$	input to a model (packet transmission rate)
$y(t)$	output from a model (packet transmission rate)
R	TCP round-trip time
$w(t)$	TCP window size
$p_{TO}(t)$	TCP timeout probability
min_{th}	minimum threshold value of RED router
max_{th}	maximum threshold value of RED router
max_p	maximum packet rejection rate of RED router
α	weight of the exponential average of RED router
$c(t)$	processing speed of RED router
$b(t)$	current queue length of RED router
$r(t)$	average queue length of RED router
$p_b(t)$	packet marking probability of RED router
$p(t)$	packet loss probability at RED router

variables) used in explanations hereafter are summarized in Table 1.

3.1 Modeling TCP Congestion Control Mechanism

In this section, the TCP congestion control mechanism is modeled as a continuous-time system where input $x(t)$ is the arrival rate for ACK packets and output $y(t)$ is the transmission rate of data packets.

In this paper, we propose a model of the TCP congestion control mechanism including its timeout mechanism by extending analytic models proposed in [7, 3, 6]. Specifically, the amount of window size decrease when a packet loss is detected using the timeout mechanism is changed from $w(t) - 1$ to $4w(t)/3 - 1$. Here, $w(t)$ denotes the TCP window size.

The reason for correcting the amount of window size decrease caused by the timeout mechanism is explained as follows. In [3, 6], the amount of window size decrease when a packet loss is detected by duplicate ACKs is given by $2w(t)/3$. This is because $w(t)$ is defined as the expected value of TCP window size. Namely, halving of TCP window size corresponds to decreasing from $4w(t)/3$ to $2w(t)/3$. However, in [6], when a packet loss is detected by the timeout mechanism, decrease of TCP window size is modeled as decreasing from $w(t)$ to 1, which is incorrect. Therefore, when a packet loss is detected by the timeout mechanism, we model that TCP window size is decreased from $4w(t)/3$ to 1.

Using fluid-flow approximation, change of TCP window size $w(t)$ including the timeout mechanism is modeled as

$$\begin{aligned} \dot{w} &= (1-q(t))\frac{w(t-R)}{w(t)R} \\ &\quad - q(t)\frac{2}{3}w(t)\frac{w(t-R)}{R}\{1-p_{TO}(t)\} \\ &\quad - q(t)\left\{\frac{4}{3}w(t)-1\right\}\frac{w(t-R)}{R}p_{TO}(t) \end{aligned} \qquad (1)$$

Here, $q(t)$ is a packet loss probability in the network. Moreover, $p_{TO}(t)$ is the probability that a packet loss is detected by the timeout mechanism rather than receiving duplicate ACKs, and can be approximated by $p_{TO}(t) \simeq \min(1, 3/w(t))$ [11]. Finally, R is a TCP connection's round-trip time.

The first term on the right side of Eq. (1) models an increase of one packet in the window size per time for one round-trip for TCP if packet loss does not occur in the network. The second term of the right side models an decrease by half in the window size if a packet loss occurs in the network and it is detected by receiving duplicate ACKs. The third term of the right side models an decrease to 1 in the window size if a packet loss occurs in the network and it is detected by the timeout mechanism.

In TCP, if packet loss does not occur in the network, the arrival rate of ACK packets will equal the previous packet transmission rate with only time for one round-trip. Accordingly, assuming $q(t)$ is the packet loss rate in the network, $x(t) = (1-q(t))y(t-R)$ is established with the arrival rate $x(t)$ for ACK packets (i.e., receiving rate by TCP). Thus, when inputs/outputs of the TCP congestion control mechanism model given by Eq. (1) are standardized to the packet transmission rate, it can be written as the following equation. Here, we use the relation $y(t) = w(t)/R$ among the TCP window size $w(t)$, the packet transmission rate $y(t)$, the round-trip time R, and $z(t) = y(t-R) - x(t)$.

$$\begin{aligned} \dot{y} &= g(x(t), y(t), R) \\ &= \frac{x(t)}{y(t)R^2} - \frac{2}{3}y(t)z(t)\{1-p_{TO}(t)\} \\ &\quad - \left\{\frac{4}{3}y(t) - \frac{1}{R}\right\}z(t)p_{TO}(t) \end{aligned} \qquad (2)$$

The following assumptions are made in the TCP model proposed in the current paper. First, all TCP connections always have data to transmit. Next, TCP round-trip time R is modeled as a constant. The TCP round-trip time is given by the sum of the round-trip propagation delay and queuing delay at the router. Thus, in actuality if the router's current queue length varies, the queuing delay at the router will also vary in response.

In Section 5, by comparing our analytic results with simulation results, we will show that the analytic model proposed in this paper has accuracy higher than the conventional analytic models.

3.2 Modeling RED router

In this section, a RED router is modeled as a continuous-time system where input $x(t)$ is the packet arrival rate and output $y(t)$ is the packet transmission rate. Note that GRED [4] is modeled in this paper.

In the conventional researches [7, 8, 3, 9], the packet marking probability determined from the average queue length of the RED router is used as the packet loss probability. However, in this paper, the packet loss probability is calculated from the packet marking probability of the RED router [5]. Namely, the packet loss probability is in fact different from the packet marking probability of the RED router.

In an actual network, there are many instances where the RED router is connected with multiple input/output links. When the RED router has multiple input links, the sum of the packet transmission rates from individual input links is $x(t)$. In addition, when the RED router has multiple output links, output $y(t)$ is distributed to multiple output links. Flow convergence and distribution like this is described in detail in Section 3.4.

The output $y(t)$ of the model of the RED router is given by the following equation.

$$\begin{aligned} y(t) &= h(x(t), p(t), c(t)) \\ &= \min(c(t), (1 - p(t))\, x(t)) \end{aligned} \tag{3}$$

Here, $p(t)$ is the packet loss probability at the RED router. The packet arrival rate at the RED router is $x(t)$, so the packet transmission rate passing through the RED router (i.e., output from the RED router) will be $(1 - p(t))x(t)$. Moreover, the limit of the packet transmission rate from the RED router is determined by the processing speed of the RED router, so the maximum value for $y(t)$ will be the processing capacity of the RED router c.

Next, derivation of the packet loss probability $p(t)$ in Eq. (3) is explained. Changes in the RED current queue length $b(t)$, average queue length $r(t)$, packet loss probability $p(t)$, and packet marking probability $p_b(t)$ are given by the following equations [7, 6]. Here, $(x)^+ \equiv \max(x, 0)$.

$$\dot{b} = \begin{cases} x(t) - c(t) & \text{if } b(t) > 0 \\ (x(t) - c(t))^+ & \text{if } b(t) = 0 \end{cases} \tag{4}$$

$$\dot{r} = -\alpha\, c(t)(r(t) - b(t)) \tag{5}$$

$$p(t) = \frac{2p_b(t)}{1 + p_b(t)} \tag{6}$$

$$p_b(t) = \begin{cases} 0 & \text{if } r(t) < min_{th} \\ \dfrac{max_p}{max_{th} - min_{th}}(r(t) - min_{th}) & \text{if } min_{th} \le r(t) < max_{th} \\ \dfrac{1 - max_p}{max_{th}} r(t) - (1 - 2max_p) & \text{if } max_{th} \le r(t) < 2max_{th} \\ 1 & \text{if } r(t) \ge 2max_{th} \end{cases} \tag{7}$$

Equation (6), with a packet marking probability of $p_b(t)$ derived from the average queue length $r(t)$, means loss of packets at the RED router with a probability of $2p_b(t)/(1 + p_b(t))$ [5]. Moreover, when the wait option of the RED router is activated, the relation between $p(t)$ and $p_b(t)$ can be expressed by $p(t) = 2p_b(t)/3$ [6].

Equation (7) indicates the packet marking probability $p_b(t)$ calculated from the average queue length $r(t)$ by the RED router. The RED router's packet marking probability $p_b(t)$ is determined by the average queue length $r(t)$ and three types of control parameters (min_{th}, max_{th}, and max_p). In addition, Gentle RED (GRED) [4] is modeled here.

With the proposed analysis technique, the packet transmission rate is reduced with each pass through the RED router in accordance with the packet loss probability. This modeling is more rigorous than the modeling techniques proposed in the literature [7, 9]. For example, an instance where two RED routers are connected is considered. At this point, the input (i.e., packet arrival rate) to the RED router in the latter stage can be suppressed by the output (i.e., packet transmission rate) from the RED router in the preceding stage. A phenomenon like this can be modeled with the analysis technique proposed in the current paper.

3.3 Modeling Link Propagation Delay

Next, the link propagation delay is modeled where the packet transmission rate input to the link is $x(t)$ and the packet transmission rate output from the link is $y(t)$.

If the propagation delay for the link is τ, $x(t)$ input to the link is delayed only by τ and output as $y(t)$. Thus, if packet loss at the link is assumed to not occur, the following equation is established between $x(t)$ and $y(t)$.

$$y(t) = x(t - \tau) \tag{8}$$

3.4 Modeling Entire Network

Next, modeling of an entire network is discussed. An entire network is modeled with the analysis technique proposed in the current paper by connection of the model of the TCP congestion control mechanism, the model of the RED router, and the model of the link propagation delay.

When this is done, a model of an entire network as modeled expresses the flow of packets in the network. In addition, a TCP feedback loop (i.e., flow of ACK packets from the receiving host to the sending host) can be expressed (Fig. 1) by input of output of a model of a RED router (placed between link models as necessary) to a TCP model. However, the following assumptions are made with respect to ACK packets for TCP. ACK packets are smaller than data packets, so congestion does not occur in the pathway that ACK packets pass through. That is, queuing delay for packets is assumed to not occur in the router that ACK packets pass through.

Next, instances where there are multiple links connected to the RED as explained simply in Section 3.2 is explained in detail. First, when the RED router has multiple input links, this is modeled as flow convergence from individual input links. Flow convergence can be described as the sum of packet transmission rates for individual links. In other words, when the transmission rate for each flow is $x_i(t)$ ($1 \leq i \leq N$) and the sum of transmission rates is $y(t)$, the following equation is established. This becomes possible because inputs/outputs for the model are all standardized to the packet transmission rate.

$$y(t) = \sum_{i=1}^{N} x_i(t) \tag{9}$$

When the RED router has multiple output links, output from the RED router is modeled through distribution in multiple flows. Flow distribution can be described by distribution of 1 packet transmission rate N times. Here, N is the number of output links. When the flow before distribution is $x(t)$, each flow after distribution is $y_i(t)$ ($1 \leq i \leq N$), and each flow distribution ratio is $f_i(t)$ ($1 \leq i \leq N$), the following equation is established.

$$y_i(t) = f_i(t)x(t) \tag{10}$$

The flow distribution ratio $f_i(t)$ ($1 \leq i \leq N$) in Eq. (10) is given by

$$f_i(t) = \frac{\sum_{s' \in S_{L(i)}} z_{s'}(t)}{\sum_{s \in S_R} z_s(t)} \tag{11}$$

Here, $S_{L(i)}$ is a set of TCP connections passing through the transmission link i, S_R is a set of TCP connections accommodated in the RED router R, and $z_s(t)$ is the packet transmission rate of the TCP connection s at time t.

4 Steady State Analysis

In what follows, using analytic models explained in Section 3, steady state analysis is performed for deriving TCP connection's throughput, the round-trip time, and the packet loss probability.

We perform steady state analysis for a general network where the number of TCP source hosts is N, and the number of RED routers is M, and the number of links is L. Moreover, a RED router has single or multiple input/output links. When a RED router has multiple output links, the output from a RED router is distributed according to the number of output links. The definition of symbols (constants and variables) used for steady state analysis is shown in Table 2.

Table 2. Definition of symbols (constants and variables) used for steady state analysis

$x_n^T(t)$	input to TCP flow n
$y_n^T(t)$	output from TCP flow n
$R_n(t)$	round-trip time of TCP flow n
$x_m^R(t)$	input to RED router m
$y_m^R(t)$	output from RED router m
$c_m(t)$	processing speed of RED router m
$q_m(t)$	current queue length of RED router m
$\overline{q}_m(t)$	average queue length of RED router m
$p_m(t)$	packet loss probability of RED router m
τ_l	propagation delay of link l
L_n	set of links that TCP flow n traverses
D_n	set of routers that TCP flow n traverses

First, from Eq. (2), the packet output rate $y_n^T(t)$ of the TCP flow n ($1 \leq n \leq N$) at time t is given by the following equation.

$$\frac{d}{dt} y_n^T(t) = g(x_n^T(t), y_n^T(t), R_n(t)) \tag{12}$$

The round-trip time $R_n(t)$ of the TCP flow n in Eq. (12) is given by the following equation.

$$R_n(t) = \sum_{l \in L_n} \tau_l + \sum_{m \in D_n} \frac{\overline{q}_m(t)}{c_m(t)} \tag{13}$$

Namely, the round-trip time of the TCP flow n is given the sum of propagation delay at all links that TCP flow n traverses, and queuing delays at all routers that the TCP flow n traverses. The queuing delay at a RED router is given by the average queue length $\overline{q}$ and the processing speed $c_m(t)$ of the RED router. Here, the processing speed $c_m(t)$ of the RED router m is assumed to be known.

Next, from Eq. (3), the packet output rate $y^R_m(t)$ of the RED router m $(1 \leq m \leq M)$ at time t is given by the following equation.

$$y^R_m(t) = h(x^R_m(t), p_m(t)) \quad (14)$$

The packet loss probability $p_m(t)$ of the RED router m in Eq. (14) is given by Eqs. (4)-(7).

From these observations, the value of each variable in steady state can be obtained by solving Eqs. (12)–(14). Let the packet output rate of the TCP flow n in steady state be y^*_n, the average queue length of the RED router m be $\overline{q}^*_m$, and the processing speed of the RED router be c^*_m. The throughput T_n and round-trip time RTT_n of the TCP flow n are given by the following equation.

$$T_n = y^*_n \quad (15)$$

$$RTT_n = \sum_{l \in L_n} \tau_l + \sum_{m \in D_n} \frac{\overline{q}^*_m}{c^*_m} \quad (16)$$

Since the throughput of the TCP flow n is the packet output rate transmitted by TCP, the throughput of the TCP flow n in steady state is given by Eq. (15). Moreover, the round-trip time of the TCP flow n is given the sum of propagation delays at all links that the TCP flow n traverses, and queuing delays at all routers that the TCP flow n traverses. Therefore, the round-trip time of the TCP flow n in steady state is given by Eq. (16).

Moreover, letting m be the packet loss probability of the RED router in steady state, the end-to-end packet loss probability P_n of the TCP flow n in steady state is given by the following equation.

$$P_n = 1 - \prod_{m \in D_n} (1 - p^*_m) \quad (17)$$

The packet loss probability p^*_m of the RED router m in steady state can be easily obtained from the average queue length $\overline{q}^*_m$ of the RED router m in steady state. The second term of the right side of Eq. (17) represents the probability that a packet loss does not occur at all RED routers that the TCP flow n traverses. Therefore, the end-to-end packet loss probability P_n of the TCP flow n can be obtained by subtracting this value from 1.

5 Numerical Examples and Simulation Results

In this section, numerical example of our analytic approach are presented. We verify the validity of our analytic approach by comparing analytic results of our steady state analysis and simulation results obtained by network simulator ns-2. Four aspects are used for comparison to the simulation results: variations in the TCP window size, variations in the RED router's current queue length, variations in the RED router's average queue length, and variations in the packet rejection rate at the RED router.

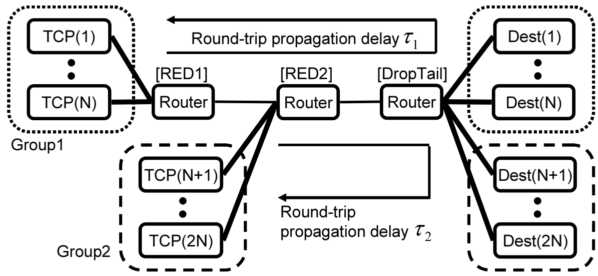

Figure 2. Network model: case of multiple TCP connections traversing different numbers of RED routers

5.1 Network Model

The network model used for verification is the case (Fig. 2) where multiple TCP connections traversing different numbers of RED routers exist. In Fig. 2, TCP (i) represents a TCP source host and Dest (i) represents a TCP destination host $(1 \leq i \leq 2N)$. Namely, data is transmitted from TCP (i) to Dest (i). Note that, in Fig. 2, the link between routers becomes the bottleneck.

Next, analytic models used for verification are continuous-time models of the TCP congestion control mechanism, the RED router (Eq. (3)–(7)), and the link propagation delay (Eq. (8)). In addition, the following three models are used as a model of the TCP congestion control mechanism. The first is an analytic model proposed in [7], which does not include a timeout mechanism. The second is an analytic model proposed in [6], which includes a timeout mechanism. The third is our analytic model (Eq. (2)), which includes a timeout mechanism. These models of the TCP congestion control mechanism are called "TCP w/o TO" (proposed in [7]), "TCP with TO1" (proposed in [6]), and "TCP with TO2" (our analytic model), respectively. Moreover, in this paper, grouping of TCP flows is performed using the same approach as [6].

First, a continuous time model of the TCP congestion control mechanism, RED router buffer, and link propagation delay explained in Section 3 is described using MATLAB/Simulink [1], which is a control engineering CAD tool. Variations in the TCP window size and the like are derived through performance of numerical simulation with respect to the continuous time model written with MATLAB/Simulink.

In addition, simulation was performed using ns-2 (version 2.27) with respect to the same network model as in

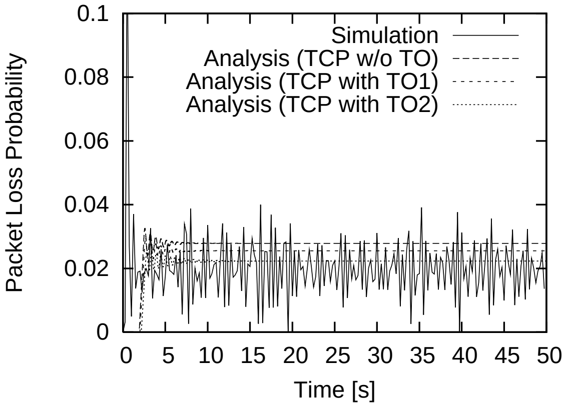

Figure 3. Analytic results and simulation results

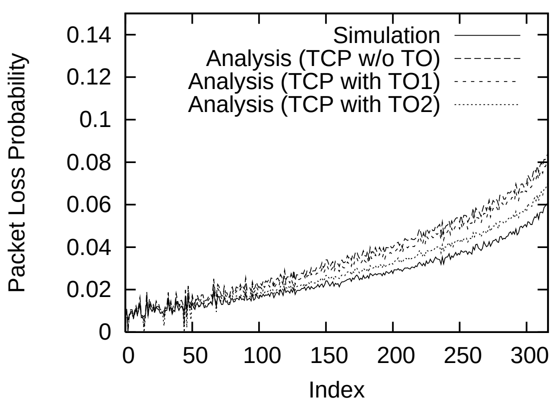

Figure 4. Analytic results and simulation results in several parameter configurations

Fig. 2. The simulation time was 50 [s], and the size of all data packets was equal at 1000 [byte]. Henceforth, analytic results and simulation results are presented for the network model.

5.2 Analytic Results and Simulation Results

The parameter configuration given to the network model is shown in Table 3. In addition, grouping of TCP flows is performed for the network model, and TCP flows are classified into two groups.

Analytic results and simulation results are shown in Fig. 3. Due to space limitation, only the evolution of the packet loss probability at the latter RED router is presented.

Packet loss probabilities of the former RED router in analytic results and simulation results were 0.

One can find that the packet loss probability of the latter RED router is different for every TCP model. Specifically, "TCP with TO2", "TCP with TO1", and "TCP w/o TO" show closer analytic results to the simulation result in this order. For instance, regarding the average queue length, the average value of simulation results is 64.880 [packet], the analytic result of "TCP with TO2" is 66.910 [packet], the analytic result of "TCP with TO1" is 69.404 [packet], and the analytic result of "TCP w/o TO" is 71.199 [packet]. Therefore, TCP with TO2 proposed in this paper shows the highest accuracy among three models.

Next, analytic results and simulation results for changing the number N of TCP flows, the processing speed c of RED routers, and the two-way propagation delay τ are shown. However, data is transmitted only from TCP connections belonging to the group 2.

We obtain analytic results and simulation results for total 350 sets of parameter configurations; the number N of TCP flows is changed to $5 + 5 \times i$ ($1 \leq i \leq 5$), the processing speed c of the RED router is changed to $0.25 + 0.25 \times j$ [packet/ms] ($1 \leq j \leq 7$), and the two-way propagation delay τ is changed to $10 \times k$ [ms] ($1 \leq k \leq 10$). In addition, values shown in Table 3 are used for parameters other than N, c, and τ. Then, the average queue length $\overline{q}$ is calculated from simulation results, and 316 of 350 analytic results, which satisfy modeling assumptions ($min_{th} \leq \overline{q} \leq 2max_{th}$), are compared with simulation results. This assumption is required since the packet loss probability of the bottleneck RED router in steady state must be positive.

Analytic results and simulation results are shown in Fig. 4. The horizontal axis of Fig. 4 is the index numbers of parameter configurations, and the vertical axis is the packet loss probability of the RED router. From Fig. 4, one can find that the accuracy of our analytic model "TCP with TO2" has the highest accuracy in various parameter configurations.

6 Summary and Future Topics

In the current paper, an analysis technique for closed-loop networks with multiple feedback loops is proposed through expansion of the analytical model proposed in the literature [7]. First, components of a network (TCP congestion control mechanism, RED router buffer, and link propagation delay) were each modeled as independent continuous-time systems using a fluid approximation model. Next, an entire closed-loop network was modeled with multistage connection of each continuous-time system. Moreover, we have proposed the analytic model of the TCP congestion control mechanism with highest accuracy. Moreover, results of steady state analysis and simulation results were compared using the analysis technique proposed and the validity of approximation analysis was veri-

Table 3. Parameters used for the network model

N	the number of TCP connections	10
τ_1	propagation delay of TCP (Group 1)	60 [ms]
τ_2	propagation delay of TCP (Group 2)	40 [ms]
c	processing space of RED router	1.5 [packet/ms]
min_{th}	minimum threshold of RED router	50 [packet]
max_{th}	maximum threshold of RED router	200 [packet]
max_p	maximum packet marking probability of RED router	0.10
α	weight of the exponential average of RED router	0.040
B	buffer size of RED router	400 [packet]

fied. Consequently, we have verified the effectiveness of our analytic approach, and have shown that our analytic model is most accurate than other conventional analytic models in various parameter configurations.

In addition, individual components of a network were modeled with continuous-time systems with the analysis technique proposed in the current paper and a model of the entire network through multistage connection of these was obtained. Various performance evaluations are thought to be possible with respect to this model through use of a control engineering CAD tool and control theory.

Unlike conventional analytic approaches, the analytic approach proposed in this paper enables modeling of a large-scale network, and several extensions and applications might be possible. Namely, we believe that our analytic approach may become a framework for performance evaluation and protocol design of a large-scale network.

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