On Transient Behavior Analysis of Random Early Detection Gateway using a Control Theoretic Approach

† Motohisa Kisimoto ‡ Hiroyuki Ohsaki ‡ Masayuki Murata

† Graduate School of Engineering Science, Osaka University
1-3 Machikaneyama, Toyonaka, Osaka 560-8531, Japan
kisimoto@ics.es.osaka-u.ac.jp

‡ Graduate School of Information Science and Technology, Osaka University
1-3 Machikaneyama, Toyonaka, Osaka 560-8531, Japan
{oosaki,murata}@ist.osaka-u.ac.jp

Abstract

AQM (Active Queue Management) mechanisms support the end-to-end congestion control mechanism of TCP (Transmission Control Protocol). Several AQM mechanisms have been recently proposed and studied by many researchers. One of popular AQM mechanisms is the RED (Random Early Detection) gateway, which randomly discards arriving packets. Although its steady state performance has been fully investigated, its transient behavior has not been well understood. In our previous work, we have analyzed the transient behavior of the RED gateway for limited cases of TCP connection variations. In this paper, by extending our previous work, we analyze the transient behavior of the RED gateway for various types of TCP connection variations (e.g., intermittent arrival or continuous arrival of multiple TCP connections). We use a control theoretic approach, which is based on the transfer function describing the relation between input and output in frequency domain. By presenting several numerical and simulation results, we discuss how control parameters of the RED gateway affect its transfer behavior.

1 Introduction

In the last few years, AQM (Active Queue Management) mechanisms that support an end-to-end congestion control mechanism of TCP have been studied by many researchers [1]. For instance, a RED (Random Early Detection) is a representative AQM mechanism, which randomly drops an arriving packet at the gateway for improving the performance to TCP traffic [2]. The authors of [2] have claimed advantages of the RED gateway over a conventional Drop-Tail gateway as follows: (1) the average queue length is kept low, (2) the performance degradation caused by a global synchronization problem found in the Drop-Tail gateway is avoided, and (3) the RED gateway improves the fairness among TCP connections. Although the effectiveness of the RED gateway is fully dependent on a choice of its four control parameters, it is not trivial to configure them appropriately.

A number of studies on the RED gateway have been extensively performed by many researchers. Most of those studies (e.g., [2, 3]) have used a simulation technique for clearly revealing characteristics of the RED gateway in various network configurations and for investigating how control parameters of the RED gateway affect its efficiency. There have been, however, a limited number of analytical studies on the RED gateway. In [4-10], the performance evaluations of the RED gateway in steady state have been performed. The authors of [2] have proposed a recommended set of control parameters, which is an empirical guideline by simulation experiments. The authors of [5, 8] have proposed another guideline, which is based on their analytic results.

Although there have been a great number of researches on the RED gateway, but most of them simply focus on its *steady state behavior*. There have been very few researches on the *transient behavior* of the RED gateway. In [11], we have analyzed the transient behavior of the RED gateway for variation in the number of TCP connections. Our analysis was based on the steady state analysis presented in [8]. To the best of our knowledge, this is the only study on the transient behavior of the RED gateway. However, in [11], we have analyzed only for temporary variation in the number of TCP connections. The transient behavior of the RED gateway for more realistic TCP connection variations has not been investigated. In this paper, we therefore analyze the transient behavior of the RED gateway for various types of TCP connections variations by extending our analytic approach presented in [11]. We use a control theoretic approach of utilizing the transfer function, which describes the relation between the input and the output in frequency domain. We also validate our approximate analysis by comparing analytic results with simulation ones.

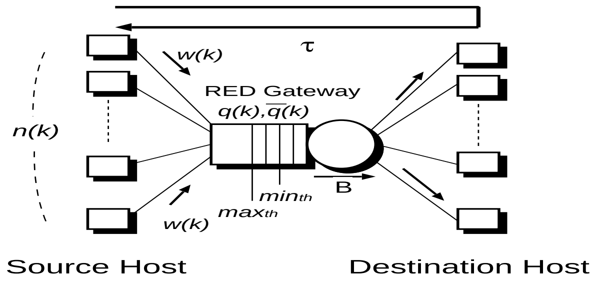

Figure 1: Analytic model.

This paper is organized as follows. In Section 2, we explain the analytic model of the RED gateway, which will be used throughout this paper. We also briefly explain the derivation of average state transition equations, which are the basis of our transient analysis. In Section 3, we analyze the transient behavior of the RED gateway for various types of TCP connection variations using a control theoretic approach. In Section 4, several numerical examples are presented to discuss how control parameters of the RED gateway affect its transient behavior. The validity of our approximate analysis is also investigated by comparing analytic results with simulation ones. In Section 5, we finally conclude this paper and discuss future works.

2 Analytic Model and State Transition Equations

In this paper, we analyze the transient behavior of the RED gateway by extending the analytic method presented in [11]. We first briefly explain our analytic model (Fig. 1) and the state transition equations derived in [11]. The definitions of symbols are summarized in Tab. 1. In [11], the following assumptions are made: (1) all TCP connections are operating in the congestion avoidance phase, (2) source hosts can detect the occurrence of a packet loss by receiving duplicate ACKs (i.e., timeout never occurs), (3) transmission speeds of all links are assumed to be sufficiently faster than the processing speed of the RED gateway, (4) all TCP connections change their window sizes synchronously, and (5) the average queue length $\overline{q}$ is between min_{th} and max_{th} because control parameters of the RED gateway are configured appropriately. Note that validity of these assumptions is discussed in [8].

Under these assumptions, the entire network is modeled by a discrete-time system, where a time slot of the system corresponds to the round-trip time of TCP connections. Namely, the derivations of the window size $w(k)$, the current queue length $q(k)$, and the average queue length $\overline{q}(k)$ are given by the following equations.

$$w(k+\overline{s}(k)) = \frac{w(k)+\overline{s}(k)-1}{2} \quad (1)$$

$$q(k+\overline{s}(k)) \simeq \frac{n(k)\,(w(k)+\overline{s}(k)-1)}{2} - B\,\tau \quad (2)$$

Table 1: Definitions of symbols.

min_{th}	minimum threshold value
max_{th}	maximum threshold value
max_p	maximum packet marking probability
q_w	weight factor for averaging
τ	propagation delay of TCP connections
B	processing speed of the RED gateway
$w(k)$	window size at slot k
$q(k)$	current queue length at slot k
$\overline{q}(k)$	average queue length at slot k
$n(k)$	the number of TCP connections at slot k

$$\overline{q}(k+\overline{s}(k)) \simeq (1-q_w)^{\overline{X}(k)}\,\overline{q}(k) + \{1-(1-q_w)^{\overline{X}(k)}\}\,q(k) \quad (3)$$

where $\overline{X}(k)$ and $\overline{s}(k)$ are given by

$$\overline{X}(k) = \frac{1}{2}\left(\frac{max_{th}-min_{th}}{max_p(\overline{q}(k)-min_{th})}+1\right)$$

$$\overline{s}(k) = \frac{1}{2} - w(k) + \frac{\sqrt{n(k)^2(1-2w(k))^2+8n(k)\overline{X}(k)}}{2n(k)}$$

Refer to [8, 11] for the detail of the derivation.

3 Transient Behavior Analysis

In what follows, we analyze the transient behavior of the RED gateway for various types of TCP connection variations by utilizing the state transition equations (Eqs. (1)–(3)). The congestion control mechanism of TCP has two operation phases: the *congestion avoidance phase* and the *slow start phase*. Similar to [11], we use two different analytic methods according to the operation phase of TCP connections.

We first classify types of TCP connection variations. Let N be the number of TCP connections established in the network. There are four types of TCP connection variations [11]:

C1: ΔN of N TCP connections terminate (or suspend) their data transmissions.

C2: ΔN TCP connections resume their data transmissions after a short idle period.

C3: ΔN TCP connections resume their data transmissions after a long idle period.

C4: ΔN TCP connections newly start their data transmissions.

In the case of C1 (or C2), $N - \Delta N$ (or $N + \Delta N$) TCP connections will operate in the congestion avoidance phase. In the cases of C3 and C4, ΔN TCP connections will operate in the slow start phase while N TCP connections will in the congestion avoidance phase.

In what follows, the equilibrium value of a variable $x(k)$ is denoted by x^*. For enabling the transient behavior analysis of the RED gateway, the state transition equations given by Eqs. (1)–(3) are linearly approximated around average equilibrium values w^*, q^*, $\overline{q}^*$, and n^*. Then, the linearized state transition equations can be written in a matrix form [8, 11]:

$$\overline{\mathbf{x}}(k + \overline{s}(k)) \simeq \mathbf{A}\overline{\mathbf{x}}(k) \tag{4}$$

where $\mathbf{A}$ is called a *state transition matrix*, and $\overline{\mathbf{x}}(k)$ is the difference between the state vector $\mathbf{x}(k)$ and the average equilibrium values. Namely, $\overline{\mathbf{x}}(k)$ is defined as

$$\overline{\mathbf{x}}(k) \equiv \begin{bmatrix} w(k) - w^* \\ q(k) - q^* \\ \overline{q}(k) - \overline{q}^* \\ n(k) - n^* \end{bmatrix}$$

As can be found from Eq. (4), the system state changes every $\overline{s}(k)$ slot, which represents the number of adjacent slots in which all packets have been unmarked by the RED gateway [8]. In what follows, we define *sequence* k as a group of $\overline{s}(k)$ slots. We also define S_k as the first slot of sequence k.

First, the cases (C1 and C2), where all TCP connections will operate in the congestion avoidance phase after the TCP connection variation, are considered. Let $u(k)$ be the difference in the numbers of TCP connections at slot $k-1$ and slot k. Also let $y(k)$ be the queue length of the RED gateway at slot k (Fig. 2). The linearized state transition equations given by Eq. (4) can be extended to include $u(k)$ and $y(k)$ as the input and the output, respectively [11]. Namely,

$$\begin{aligned} \overline{\mathbf{x}}(k + \overline{s}(k)) &= \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) \\ y(k) &= \mathbf{C}\overline{\mathbf{x}}(k) \\ \mathbf{B} &= [\,0\,0\,0\,1\,]^T \\ \mathbf{C} &= [\,0\,1\,0\,0\,] \end{aligned} \tag{5}$$

Second, the other two cases (C3 and C4), where a part of TCP connections will operate in the slow start phase after the TCP connection variation, are considered. Let $u(k)$ be the difference in the total numbers of packets sent from source hosts operating in the slow start phase at slot $k-1$ and slot k. Let $y(k)$ be the queue length of the RED gateway at slot k (Fig. 3). Similar to the previous cases, the linearized state transition equations given by Eq. (4) can be extended to include $u(k)$ and $y(k)$ as the input and the output, respectively.

$$\begin{aligned} \overline{\mathbf{x}}(k + \overline{s}(k)) &= \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) \\ y(k) &= \mathbf{C}\overline{\mathbf{x}}(k) \end{aligned} \tag{6}$$

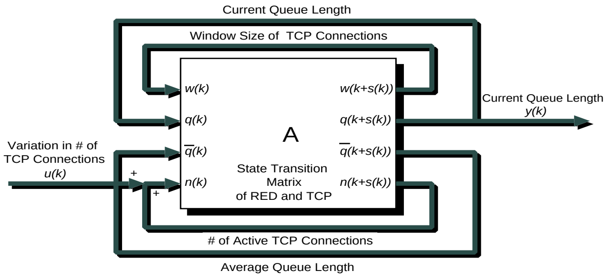

Figure 2: Cases C1 and C2 where all TCP connections in congestion avoidance phase.

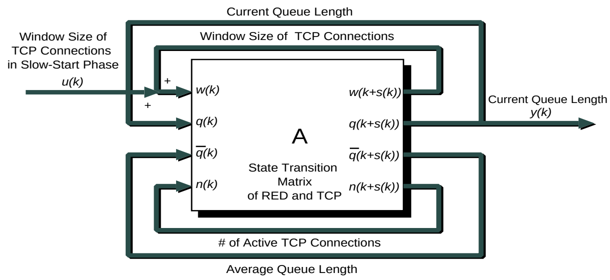

Figure 3: Cases C3 and C4 where a part of TCP connections in slow start phase.

$$\begin{aligned} \mathbf{B} &= [\,1\,0\,0\,0\,]^T \\ \mathbf{C} &= [\,0\,1\,0\,0\,] \end{aligned}$$

In this paper, we analyze the transient behavior of the RED gateway for various types of TCP connection variations by utilizing the transfer function. The transfer function of a linear system describes the relation between the input and the output in frequency domain [12]. We define z-transforms of the input $u(k)$ and the output $y(k)$ as $U(z)$ and $Y(z)$, respectively. The transfer function $G(z)$ of a system satisfies the following relation.

$$Y(z) = G(z)\,U(z) \tag{7}$$

The transfer functions of the system described by Eqs. (5) and (6) are given by the following equation [12].

$$G(z) = \mathbf{C}(z\,\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}$$

It is known that the stability and the transient behavior of a closed-loop system are determined by the poles of its transfer function. By investigating the modulus of poles λ_i, the stability and the transient behavior of the system can be easily known. In our transient behavior analysis, the input is the z-transform of either the difference in the numbers of TCP connections (C1 and C2) or the difference in the sum of window sizes of all source hosts (C3 and C4). The transient performance of the RED gateway is therefore determined by poles of $G(z) \times U(z)$. However, Eq. (7) always has the modulus of 1.0 since the number of TCP connections $n(k)$ is changed only by the input $U(z)$. It is therefore necessary to exclude the modulus of 1.0 when analyzing the transient behavior since the pole corresponding to the modulus of 1.0 has no impact on the transient behavior of the queue length of the RED gateway. Note that we have linearly approximated the system around its equilibrium values when deriving Eq. (4). It is therefore expected that the approximation error becomes large when the value of the window size or the queue length deviates from their equilibrium values. In Section 4, we therefore validate our approximate analysis by comparing analytic results with simulation ones.

First, we explain examples of the input $U(z)$ in cases C1 and C2, where all TCP connections will operate in the congestion avoidance phase.

1. Case of temporary change in the number of TCP connections

   A temporary change in the number of TCP connections can be formulated by an impulse. For example, when the number of TCP connections is increased by ΔN (or decreased if ΔN is negative) at sequence i, the input $u(k)$ and its z-transform $U(z)$ are given by

$$u(k) = \begin{cases} \Delta N & \text{if } k = S_i \\ 0 & \text{otherwise} \end{cases}$$
$$U(z) = \Delta N \, z^{-i} \quad (8)$$

2. Case of generic change in the number of TCP connections

   A generic change in the number of TCP connections can be given by the convolution of multiple impulses. For instance, when the number of TCP connections is increased (or decreased) by ΔN_i at sequence t_i, the z-transform of the input, $U(z)$, is given by

$$U(z) = \sum_i \Delta N_i z^{-t_i} \quad (9)$$

3. Case of continuous change in the number of TCP connections

   A continuous change in the number of TCP connections can be formulated by the step function. For example, when the number of TCP connections is increased (or decreased) by ΔN at each sequence, the z-transform of the input, $U(z)$, is given by

$$U(z) = \frac{\Delta N \, z}{z - 1}$$

Second, we explain examples of the input $U(z)$ in cases C3 and C4, where a part of TCP connections will operate in the slow start phase.

1. Case of temporary increase in the number of TCP connections

   When a part of TCP connections will operate in the slow start phase, window sizes of these connections are doubled every round-trip time. Hence, when the number of TCP connections in the slow start phase is increased by ΔN at sequence i, $u(k)$ and $U(z)$ are approximately given by

$$u(k) \simeq \begin{cases} \frac{\Delta N}{n(k)} \times 2^{\overline{s}(k)(k - S_i - 1)} & \text{if } k > S_i \\ 0 & \text{otherwise} \end{cases}$$
$$U(z) \simeq \frac{2^{\overline{s}^*(-S_i-1)} \, \Delta N \, z}{n^* \, (z-2)} \quad (10)$$

   In the above equation, we approximate $n^* \equiv n(k)$ since the number $n(k)$ of TCP connections in the congestion avoidance phase does not change. Similarly, we approximate $\overline{s}^* \equiv \overline{s}(k)$. Note that in Eq. (10) we assume that ΔN TCP connections established at sequence i will not operate in the congestion avoidance phase at slot k ($k > S_i$).

2. Case of generic increase in the number of TCP connections

   A generic increase in the number of TCP connections can be formulated by the convolution of Eq. (10). For instance, when the number of TCP connections operating in the slow start phase is increased by ΔN_i at sequence t_i, z-transform $U(z)$ is given by

$$U(z) = \sum_i \frac{\Delta N_i \, 2^{\overline{s}^*(-S_i-1)} \, z^{-t_i+1}}{n^* \, (z-2)}$$

4 Numerical Examples and Discussions

In this section, by presenting several numerical examples, we discuss the relation between a choice of control parameters of the RED gateway and its transient behavior. We also validate our approximate analysis by comparing analytic results with simulation ones. In all analytic and simulation results, without explicitly stated, the following parameters are used: the number of TCP connections in steady state $n^* = 5$, the processing speed of the RED gateway $B = 0.2$ [packet/ms], the propagation delay $\tau = 1$ [ms]. Four control parameters of the RED gateway are configured according to the recommendation in [2], i.e., $min_{th} = 5$, $max_{th} = 15$, $max_p = 0.1$ and $q_w = 0.002$.

Figure 4 shows the stability region when the number of TCP connections, operating in the congestion avoidance phase, is incremented by one. This figure shows the maximum modulus of poles of $U(z) \times G(z)$ (Eq. (7)) in the min_{th}–max_{th} plane when the input $U(z)$ is given by Eq. (8) with $i = 0$. This figure shows that the operation of the RED gateway becomes unstable (i.e., the queue length of the RED gateway never converges) when the maximum modulus of poles is larger than 1.0. This figure also shows that the smaller the maximum modulus of poles is, the better the transient behavior of the RED gateway becomes. One can find from this

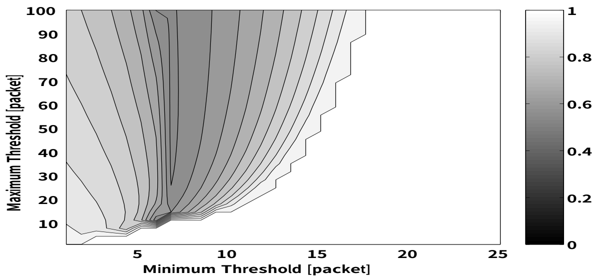

Figure 4: The maximum modulus of poles in the min_{th}–max_{th} plane.

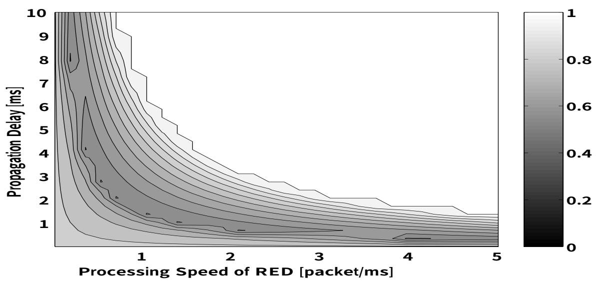

Figure 5: The maximum modulus of poles in the B–τ plane.

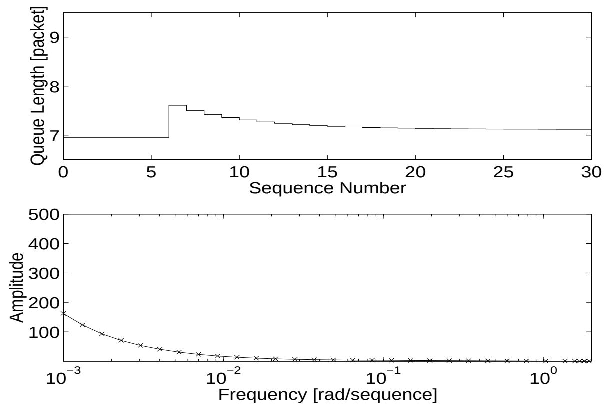

Figure 6: Impulse response and gain characteristic of $U(z) \times G(z)$ for $\Delta N_1 = 1$.

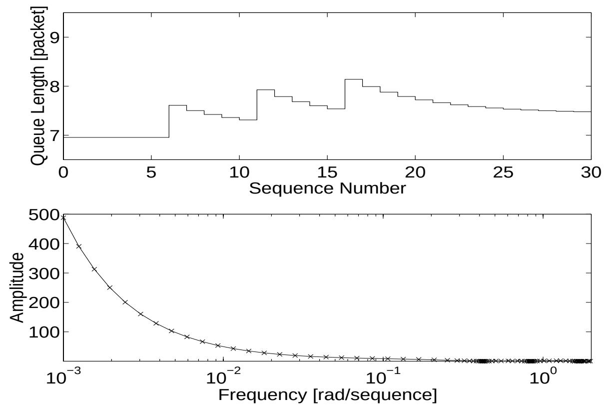

Figure 7: Impulse response and gain characteristic of $U(z) \times G(z)$ for $(\Delta N_1, \Delta N_2, \Delta N_3) = (1, 1, 1)$.

figure that the control parameter min_{th} has a large impact on the transient behavior of the RED gateway. For instance, in this case, the transient behavior of the RED gateway is optimal if min_{th} is chosen around 8.0. On the contrary, one can also find that the control parameter max_{th} has little impact on the stability and the transient behavior. Note that similar tendency is discovered in [8], where the steady state analysis of the RED gateway is performed. From these observations, we can conjecture that if control parameters of the RED gateway are configured for optimizing its steady state performance, the transient behavior is also optimized.

In Fig. 5, we next show the stability region for $min_{th} = 5$ and $max_{th} = 15$. In this figure, the maximum modulus of poles of $U(z) \times G(z)$ is plotted in the B–τ plane. This figure indicates that both the processing speed of the RED gateway B and the propagation delay τ affect the transient behavior of the RED gateway. This figure also indicates that the maximum modulus of poles is mostly determined by the bandwidth–delay product (i.e., $B \times \tau$).

Figures 6 through Fig. 8 present numerical results from our transient behavior analysis based on the transfer function. These figures are for the cases that the number of TCP connections, operating in the congestion avoidance phase, is increased. In all these figures, Eq. (9) is used as the input $U(z)$, but t_i's and ΔN_i's are set to different values. In Fig 6, $t_1 = 5$ and $\Delta N_1 = 1$ are used for analyzing the case that the number of TCP connections is incremented by one. In Fig. 7, $(t_1, t_2, t_3) = (5, 10, 15)$ and $(\Delta N_1, \Delta N_2, \Delta N_3) = (1, 1, 1)$ are used for analyzing the case that the number of TCP connections is incremented by three. Finally, in Fig. 8, $(t_1, t_2, t_3) = (5, 10, 15)$ and $(\Delta N_1, \Delta N_2, \Delta N_3) = (1, 2, 1)$ for analyzing the case that the number of TCP connections is incremented by four.

In each figure, the impulse response of the transfer function $U(z) \times G(z)$ (upper) and the gain characteristic of $U(z) \times G(z)$ (lower). The impulse response illustrates the evolution of the queue length of the RED gateway. The gain characteristic illustrates the amplitude of the transfer function $U(z) \times G(z)$ at different frequencies. Note that these impulse responses are obtained not from iterative computation using Eq. (4), but from direct calculation using the transfer function and the MATLAB language. By analyzing these impulse responses, we can investigate the transient behavior of the RED gateway for various types of TCP connection variations. For example, comparing gain characteristics of Figs. 7 and 8 tells us that the amplitude of the queue length of the RED gateway in Fig. 8 is larger than that in Fig. 7. In other words, using the transfer function enables us to analyze the transient behavior of the RED gateway not in time domain but in frequency domain.

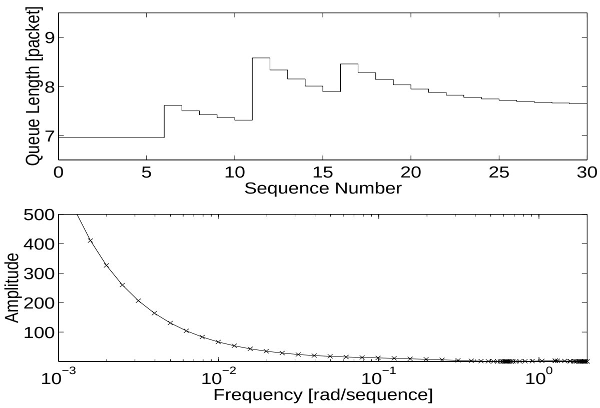

Figure 8: Impulse response and gain characteristic of $U(z) \times G(z)$ for $(\Delta N_1, \Delta N_2, \Delta N_3) = (1, 2, 1)$.

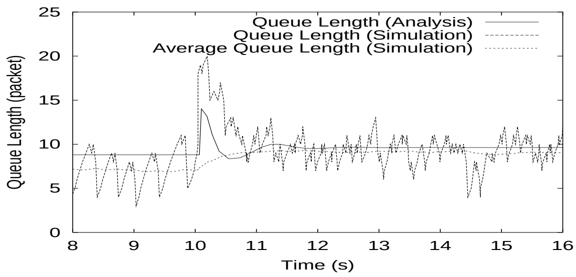

Figure 9: Comparison between analytic and simulation results.

Finally, in Fig. 9, we show a simulation result when all TCP connections operate in the congestion avoidance phase (i.e., case C2). This figure shows the evolution of the queue length when the number of TCP connections operating in the congestion avoidance phase is incremented by one at $t = 10$ [s]. In this simulation, we use the ns-2 simulator for the same network model with Fig. 1. The size of all TCP packets is set to 1,000 [bytes]. In this figure, the analytic result is also plotted for comparison purposes. This figure shows that the analytic result roughly coincides to the simulation result. In particular, after the change in the number of TCP connections, the settling time of the queue length in simulation is almost identical to the analytic result. However, one can find that the amplitude of the queue length in our analysis is smaller than that of simulation. Such difference might be caused by approximation errors in linearization or conservative estimation of $w(k)$ or $q(k)$. As a future work, more investigation for minimizing the difference between analytic and simulation results would be appropriate.

5 Conclusion and Future Work

In this paper, we have analyzed the transient behavior of the RED gateway for various types of TCP connection variations such as intermittent arrival or continuous arrival of multiple TCP connections. We have used a control theoretic approach, which is based on the transfer function describing the relation between input and output not in time domain but in frequency domain. Using the transfer function, various characteristics of a feedback system can be investigated. We are currently working to extend our analytic approach for other AQM mechanisms.

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