Steady State Analysis of TCP Connections with Different Propagation Delays

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1 Introduction

In our previous work [1], we have proposed an approach for modeling the network including the TCP mechanism running on a source host as a single feedback system. Our approach was to separately model the TCP congestion control mechanism and the network using the fluid flow approximation and the queuing theory, respectively. We have modeled the congestion control mechanism of TCP as a SISO (Single Input and Single Output) system, where the input to the system is an observed packet loss probability in the network and the output is a TCP window size. We have modeled the network seen by TCP as another SISO system, where the input to the system is the TCP window size and the output is the packet loss probability in the network. In [1], our steady state analysis was limited to a network with either a single TCP connection or homogeneous TCP connections (i.e., TCP connections with identical propagation delays).

In this paper, using a similar modeling approach proposed in [1], we analyze the network with multiple TCP connections, where each TCP connection is allowed to have different propagation delay. We first present the steady state analysis for a network with a single TCP connection, and derive the TCP throughput, the TCP round-trip time, and the packet loss probability in the network. We then extend it to a network with heterogeneous TCP connections.

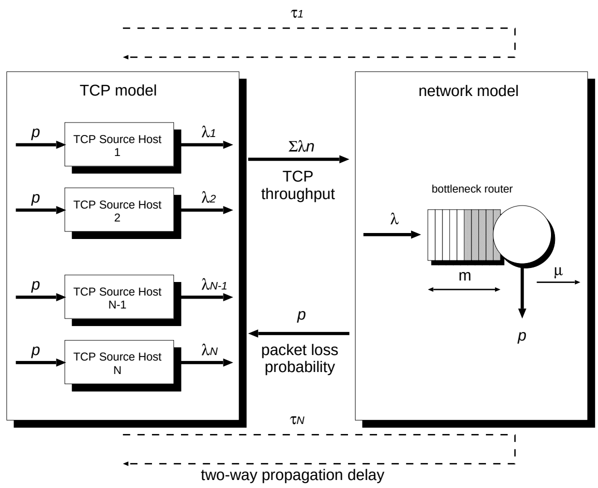

Figure 1: Analytic model.

Table 1 Definition of symbols.

N	the number of TCP connections
μ	packet processing speed of bottleneck router
m	buffer size of bottleneck router
τ_n	two-way propagation delay of nth TCP connection
r_n	round-trip time of nth TCP connection
λ_n	throughput of nth TCP connection
p	packet loss probability in the network

2 Analytic Model

Figure 1 illustrates our analytic model. The number N of TCP connections share the single bottleneck link, and each TCP connection has a different propagation delay τ_n. We model the entire network, including TCP mechanisms running on source hosts, as a single feedback system, where the congestion control mechanism of TCP and the network seen by TCP interact each other. Table 1 summarizes the definition of symbols used throughout this paper. Note that the TCP throughput λ_n is defined as the packet transmission rate from the source host, and is different from the TCP goodput.

3 Steady State Analysis

3.1 Case of a single TCP connection

We first present the steady state analysis for the network with a single TCP connection. The TCP source host is modeled by a SISO (Single Input and Single Output) system; i.e., the packet loss probability p in the network is the input to the system, and the TCP throughput λ is the output. Let p and r be the packet loss probability in the network and the round-trip time of the TCP connection, respectively. The TCP throughput $\lambda(p, r)$ is approximately given by the following equation [2].

$$\lambda(p, r) \simeq \frac{1}{r}\sqrt{\frac{3}{2\,b\,p}} \qquad (1)$$

In the above equation, b (usually $b = 1$ or $b = 2$) is the number of packets required for the TCP destination host to send an ACK packet.

We then model the network seen from the TCP source host by a single FIFO (First-In First-Out) queue. Provided that queueing delay occurs only at the bottleneck router, the bottleneck router is modeled by a M/M/1 queuing system. In other words, the network is modeled by a SISO system, where the input is the TCP throughput λ and the output is the packet loss probability p at the bottleneck router. Let λ and m be the TCP throughput and the buffer size of the bottleneck router. The packet loss probability in the network is approximated as

$$p(\lambda) \simeq 1 - \sum_{k=0}^{m} P_k(\lambda) \qquad (2)$$

where $P_k(\lambda)$ is the state probability of a M/M/1 queuing system. By letting μ be the packet processing speed of the bottleneck router, $P_k(\lambda)$ is given by

$$P_k(\lambda) \quad = \quad \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k \tag{3}$$

Thus, $p(\lambda)$ is obtained from Eqs. (2) and (3) as

$$p(\lambda) \quad \simeq \quad \left(\frac{\lambda}{\mu}\right)^{1+m} \tag{4}$$

Since the window-based flow control mechanism of TCP operates around the 100% offered traffic load, $p(\lambda)$ in the above equation can be approximated by

$$p(\lambda) \quad \simeq \quad \frac{\lambda + \lambda\, m - m\, \mu}{\mu} \tag{5}$$

We then derive the round-trip time of the TCP connection. Let τ be the propagation delay of a TCP connection. We assume that the queueing delay occurs only at the bottleneck router. For a given TCP throughput λ, the TCP round-trip time $r(\lambda)$ is obtained using the average queue length $L = \lambda/(\mu - \lambda)$ of a M/M/1 queueing system and the Little's theorem $N = \lambda T$. Thus,

$$r(\lambda) \quad = \quad \frac{L}{\lambda} + \tau = \frac{1}{\mu - \lambda} + \tau \tag{6}$$

Let λ^*, r^*, and p^* be the TCP throughput, the round-trip time of the TCP connection, and the packet loss probability in the network in steady state, respectively. From Eqs. (1), (5), and (6), the following relations are satisfied in steady state.

$$\begin{aligned} \lambda(p^*, r^*) &= \lambda^* \\ p(\lambda^*) &= p^* \\ r(\lambda^*) &= r^* \end{aligned}$$

By solving these equations, we can obtain the TCP throughput λ^*, the round-trip time r^* of the TCP connection, and the packet loss probability p^* in steady state. For example, the TCP throughput λ^* in steady state is given by the solution of the following equation.

$$\frac{(\mu - \lambda)\, \sqrt{\frac{3\,\mu}{2\,b\,(\lambda + \lambda\, m - m\,\mu)}}}{1 - \lambda\,\tau + \mu\,\tau} = \lambda \tag{7}$$

3.2 Case of multiple TCP connections

In what follows, we extend the previous analysis to a network with heterogeneous TCP connections. Let N be the number of TCP connections, λ_n be the throughput of nth TCP connection, and r_n be the round-trip time of nth TCP connection. We assume random packet losses in the network. From Eq. (1), for a given packet loss probability p in the network, the throughput of each TCP connection is given by

$$\lambda_n(p, r_n) \quad \simeq \quad \frac{1}{r_n}\sqrt{\frac{3}{2\,b\,p}} \tag{8}$$

We then model the network seen from TCP source hosts as a MISO (Multi Input and Single Output) system, where the inputs are throughputs λ_n of TCP connections and the output is the packet loss probability p in the network. From Eq. (2), the packet loss probability in the network with multiple TCP connections, $p(\lambda_1, \cdots, \lambda_N)$, is obtained as

$$p(\lambda_1, \cdots, \lambda_N) \quad \simeq \quad 1 - \sum_{k=0}^{m} P_k\left(\sum_{n=1}^{N} \lambda_n\right) \tag{9}$$

Similarly, for a given TCP throughput λ_n, the round-trip time $r_n(\lambda_n)$ of nth TCP connections is obtained from Eq. (6).

$$r_n(\lambda_n) \quad = \quad \frac{1}{\mu - \sum_{n=1}^{N} \lambda_n} + \tau_n \tag{10}$$

where τ_n is the propagation delay of nth TCP connection.

Finally, we derive the equilibrium in steady state. Let λ_n^*, r_n^*, and p^* be the throughput of nth TCP connection, the round-trip time of nth TCP connection, and the packet loss probability in steady state, respectively. We have the following equations from Eqs. (8), (9), and (10).

$$\begin{aligned} \lambda_1(p^*, r_1^*) &= \lambda_1^* \\ &\vdots \\ \lambda_N(p^*, r_N^*) &= \lambda_N^* \\ p(\lambda_1^*, \cdots, \lambda_N^*) &= p^* \\ r(\lambda_1^*) &= r_1^* \\ &\vdots \\ r(\lambda_N^*) &= r_N^* \end{aligned}$$

By numerically solving these equations for λ_n^*, r_n^*, and p^*, the throughput and the round-trip time of nth TCP connection, and the packet loss probability in the network can be obtained.

4 Conclusion

In this paper, we have analyzed the steady state behavior of TCP in a network with heterogeneous TCP connections, where each TCP connection has a different propagation delay. As a future work, it would be interesting to apply our analytic method to more general network models, for example, where several bottleneck routers exist in the network.

Refernces

[1] H. Hisamatu, H. Ohsaki, and M. Murata, "Steady state and transient behavior analyses of TCP connections considering interactions between TCP connections and network," submitted to *IEEE GLOBECOM2002 (GLOBE2002)*, Mar. 2002.

[2] J. Padhye, V. Firoiu, D. Towsley, and J. Kurose, "Modeling TCP throughput: a simple model and its empirical validation," in *Proceedings of ACM SIGCOMM '98*, pp. 303–314, 1998.